On the Rainbow Vertex Connection Number of Edge Comb of Some Graph

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Abstract—By an edge comb, we mean a graph formed by combining two graphs G and H, where each edge of graph G is replaced by the which one edge of graph H, denote by $G \trianglerighteq H$. A vertex colored graph $G \trianglerighteq H = (V(G \trianglerighteq H), E(G \trianglerighteq H))$ is said rainbow vertex-connected, if for every two vertices u and v in $V(G \trianglerighteq H)$, there is a $u - v$ path with all internal vertices have distinct color. The rainbow vertex connection number of $G \trianglerighteq H$, denoted by $rvc(G \trianglerighteq H)$ is the smallest number of color needed in order to make $G \trianglerighteq H$ rainbow vertex-connected. This research aims to find an exact value of the rainbow vertex connection number of exponential graph, namely $rvc(G \trianglerighteq H)$ when $G \trianglerighteq H$ are $P_n \trianglerighteq Bt_m$, $S_n \trianglerighteq Bt_m$, $L_n \trianglerighteq Bt_m$, $F_{m,n} \trianglerighteq Bt_p$, $rvc(P_n \trianglerighteq S_m)$, $rvc(C_n \trianglerighteq S_m)$, and $rvc(W_n \trianglerighteq S_m)$ $W_n \trianglerighteq Bt_m$. The result shows that the resulting rainbow vertex connection attain the given lower bound.

Keywords—Rainbow vertex connection coloring, rvc number, edge comb.

INTRODUCTION

Rainbow vertex connection concept was first introduced in 2009 by Krivelevich and Yuster [1]. These new concept arised from information exchange interconnection communication of information between agencies and government. When we need to route a messages in a cellular network in such a way that each link on the route between two vertices is assigned with a distinct channel such a problem is considered to be a rainbow colour. The minimum number of channels that we have to use, is exactly a rainbow connection number. For more details various rainbow connections we refer to [2] [3] [4] [5] [6] [7] [8] [9].

Suppose G is a connected graph *nontrivial* with *vertex-coloring* $c : V(G) \to \{1, 2, 3, \ldots, n\}$, $n \in \mathbb{N}$, that vertex adjacent may have same color, we refer to [10] [11] [12]. A vertex-colored graph is rainbow vertex-connected if any two vertices are connected by a path whose internal vertex have distinct colors. The *rainbow vertex connection* of a connected graph G, denote by $rvc(G)$, is the smallest number of colors that are needed in order to make G rainbow vertex-connected, we refer to [13].

A graph formed by combining two graphs, suppose graph G and H, where each edge of graph G is replaced with which one edge of graph H is called comb product, denote by $G \trianglerighteq H$. If $|V(G)| = p_1$ and $|E(G)| = q_1$, while $|V(H)| = p_2$ and $|E(H)| = q_2$ then for $|V(G \trianglerighteq H)| = q_1(p_2 - 2) + p_1$ and $|E(G \trianglerighteq H)| = q_1 q_2$.

The Research activities on rainbow vertex connection is growing rapidly, we refer the result to [14] [15]. Simamora [14] show that.

Theorem 1. *[14]If $n \geq 2$, then*

$$rvc(Pc_n) = \begin{cases} \lceil \frac{n}{2} \rceil; & \text{for } n \leq 7, \\ \lceil \frac{n}{2} \rceil + 1; & \text{for another } n \end{cases}$$

Theorem lower bound of *rainbow vertex connection number* is shown by [1]

Theorem 2. *For any graph G,*

$$rvc(G) \geq diam(G) - 1$$

THE RESULTS

The followings show rainbow vertex connection number $rvc(G)$ when G are $P_n \trianglerighteq Bt_m$, $S_n \trianglerighteq Bt_m$, $L_n \trianglerighteq Bt_m$, $F_{m,n} \trianglerighteq Bt_p$, and $W_n \trianglerighteq Bt_m$.

◊ **Theorem 1.** *Let G be a comb product denote by $P_n \trianglerighteq Bt_m$, for $n \geq 3$ and $m \geq 2$. The rainbow vertex connection number $rvc(P_n \trianglerighteq Bt_m) = n - 2$*

Proof. Let G be a comb product denote by $P_n \trianglerighteq Bt_m$. The vertex set of G is $V(P_n \trianglerighteq Bt_m) = \{x_i; 1 \leq i \leq n\}$ $\cup \{x_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\}$ and the edge set is $E(P_n \trianglerighteq Bt_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i x_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{x_{i+1} x_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\}$. The order of the graph $|V(P_n \trianglerighteq Bt_m)| = n + mn - m$ and the size is $|E(P_n \trianglerighteq Bt_m)| = n - 2m + 2mn - 1$. According Theorem 2 the lower bound are stated as follow: $rvc(P_n \trianglerighteq Bt_m) \geq diam(P_n \trianglerighteq Bt_m) - 1$, where $diam(P_n \trianglerighteq Bt_m) = n - 1$ is the diameter of graph G. Since $diam(P_n \trianglerighteq Bt_m) = n - 1$, for $n \geq 3$ and $m \geq 2$, it follows that $rvc(P_n \trianglerighteq Bt_m) \geq n - 1 - 1$. And than, will be proof that $rvc(P_n \trianglerighteq Bt_m) \leq n - 2$ by construct $c : V(G) \to \{1, 2, 3, \ldots, k\}$ as follows:

$$c(v) = \begin{cases} 1, & \text{for } v = x_i;\ i = 1, n \\ i - 1, & \text{for } v = x_i;\ 2 \leq i \leq n - 1 \\ 1, & \text{for } v = x_{i,j};\ 1 \leq i \leq n, \\ & 1 \leq j \leq m \end{cases}$$

From the function can determine $rvc(P_n \trianglerighteq Bt_m) \leq n - 2$. There for $rvc(P_n \trianglerighteq Bt_m) \geq n - 2$ and $rvc(P_n \trianglerighteq Bt_m) \leq n - 2$, then $rvc(P_n \trianglerighteq Bt_m) = n - 2$. □

◊ **Theorem 2.** *Let G be a comb product denote by $S_n \trianglerighteq Bt_m$, for $n \geq 3$ and $m \geq 2$. The rainbow vertex connection number $rvc(S_n \trianglerighteq Bt_m) = 1$.*

Proof. Let G be a comb product denote by $S_n \trianglerighteq Bt_m$. The vertex set of G is $V(S_n \trianglerighteq Bt_m) = \{A\}$ $\cup \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge set is $E(S_n \trianglerighteq Bt_m) = \{A x_i; 1 \leq i \leq n\}$ $\cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$. The order of the graph $|V(S_n \trianglerighteq Bt_m)| = mn + n + 1$ and the size is $|E(S_n \trianglerighteq Bt_m)| = 2mn + n$. According Theorem 2 and lower bound are stated as follow: $rvc(S_n \trianglerighteq Bt_m) \geq diam(S_n \trianglerighteq Bt_m) - 1$, where $diam(S_n \trianglerighteq Bt_m)$ is the diameter of graph G. Since $diam(S_n \trianglerighteq Bt_m) = 2$, for $n \geq 3$ dan $m \geq 2$, it follows that $rvc(S_n \trianglerighteq Bt_m) \geq 2 - 1$. And than, will be proof that $rvc(S_n \trianglerighteq Bt_m) \leq 1$ by construct $c : V(G) \to \{1, 2, 3, \ldots, k\}$ as follows:

$$c(v) = \begin{cases} 1, & \text{for } v = A \\ 1, & \text{for } v = x_i;\ 1 \leq i \leq n \\ 1, & \text{for } v = x_{i,j};\ 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

From the function can determine $rvc(S_n \trianglerighteq Bt_m) \leq 1$. Therefore $rvc(S_n \trianglerighteq Bt_m) \geq 1$ and $rvc(S_n \trianglerighteq Bt_m) \leq 1$, then $rvc(S_n \trianglerighteq Bt_m) = 1$. □

♢ **Theorem 3.** *Let G be a comb product denote by $L_n \trianglerighteq Bt_m$, for $n \geq 3$ and $m \geq 2$. The rainbow vertex connection number $rvc(L_n \trianglerighteq Bt_m) = n$.*

Proof. Let G be a comb product denote by $L_n \trianglerighteq Bt_m$. The vertex set of G is $V(L_n \trianglerighteq Bt_m) = \{x_i, y_i; 1 \leq i \leq n\} \cup \{x_{i,j}, y_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{z_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge set is $E(L_n \trianglerighteq Bt_m) = \{x_i x_{i+1}, y_i y_{i+1}; 1 \leq i \leq n\} \cup \{x_i y_j; 1 \leq i \leq n\} \cup \{x_i x_{i,j}, y_i y_{i,j}, x_{i+1} x_{i,j}, y_{i+1} y_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{x_i z_{i,j}, y_i z_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$. The order of the graph $|V(L_n \trianglerighteq Bt_m)| = 3mn - 2m + 2n$ and the size is $|E(L_n \trianglerighteq Bt_m)| = 6mn - 4m + 3n - 2$, dan diameter $diam(L_n \trianglerighteq Bt_m) = n + 1$. According Theorem 2 the lower bound are stated as follow: $rvc(L_n \trianglerighteq Bt_m) \geq diam(L_n \trianglerighteq Bt_m) - 1$, where $diam(L_n \trianglerighteq Bt_m)$ is the diameter of graph G. Since $diam(L_n \trianglerighteq Bt_m) = n + 1$, for $n \geq 3$ dan $m \geq 2$, it follows that $rvc(L_n \trianglerighteq Bt_m) \geq n + 1 - 1$. And than, will be proof that $rvc(L_n \trianglerighteq Bt_m) \leq n$ by construct $c : V(G) \rightarrow \{1, 2, 3, \ldots, k\}$ as follows:

$$c(v) = \begin{cases} 1, & \text{for } v = x_{i,j}, y_{i,j}; \\ & 1 \leq i \leq n-1, 1 \leq j \leq m \\ 1, & \text{for } v = z_{i,j}; 1 \leq i \leq n \\ & 1 \leq j \leq m \\ i, & \text{for } v = x_i; 1 \leq i \leq n \\ n-1, & \text{for } v = y_i; 1 \leq i \leq n \end{cases}$$

From the function can determine $rvc(L_n \trianglerighteq Bt_m) \leq n$. Therefore $rvc(L_n \trianglerighteq Bt_m) \geq n$ and $rvc(L_n \trianglerighteq Bt_m) \leq n$, then $rvc(L_n \trianglerighteq Bt_m) = n$. □

♢ **Theorem 4.** *Let G be a comb product denote by $F_{m,n} \trianglerighteq Bt_p$, for $m \geq 3$, $n \geq 2$, and $p \geq 2$. The rainbow vertex connection number $rvc(F_{m,n} \trianglerighteq Bt_p) = 3$.*

Proof. Let G be a comb product denote by $F_{m,n} \trianglerighteq Bt_p$. The vertex set of G is $V(F_{m,n} \trianglerighteq Bt_p) = \{A\} \cup \{x_{i,j}; 1 \leq i \leq m, 1 \leq j \leq m\} \cup \{x_{i,j,k}; 1 \leq i \leq m, 1 \leq j \leq n-1, 1 \leq k \leq p\} \cup \{y_{i,j,k}; 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq p\}$ and the edge set is $E(F_{m,n} \trianglerighteq Bt_p) = \{A x_{i,j}; 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{A y_{i,j,k}; 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq p\} \cup \{x_{i,j} y_{i,j,k}; 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq p\} \cup \{x_{i,j} x_{i,j,k}; 1 \leq i \leq m, 1 \leq j \leq n-1, 1 \leq k \leq p\} \cup \{x_{i,j+1} x_{i,j,k}; 1 \leq i \leq m, 1 \leq j \leq n-1, 1 \leq k \leq p\}$. The order of the graph $|V(F_{m,n} \trianglerighteq Bt_p)| = 2mnp + mn - p + 1$ and the size is $|E(F_{m,n} \trianglerighteq Bt_p)| = 4mnp + 2mn - 2mp - m$. According Theorem 2 the lower bound are stated as follow: $rvc(F_{m,n} \trianglerighteq Bt_p) \geq diam(F_{m,n} \trianglerighteq Bt_p) - 1$, where $diam(F_{m,n} \trianglerighteq Bt_p)$ is the diameter of graph G. Since $diam(F_{m,n} \trianglerighteq Bt_p) = 4$, for $m \geq 3$, $n \geq 2$, and $p \geq 2$, it follows that $rvc(F_{m,n} \trianglerighteq Bt_p) \geq 4 - 1$. And than, will be proof that $rvc(F_{m,n} \trianglerighteq Bt_p) \leq 3$ by construct $c : V(G) \rightarrow \{1, 2, 3, \ldots, k\}$ as follows:

$$\begin{cases} 1, & \text{untuk } v = A, x_{i,j,k}; 1 \leq i \leq m, \\ & 1 \leq j \leq n-1, 1 \leq k \leq p \\ 1, & \text{untuk } v = y_{i,j,k}; 1 \leq i \leq m, \\ & 1 \leq j \leq n, 1 \leq k \leq p \\ 2, & \text{untuk } v = x_{i,2j-1}; 1 \leq i \leq m, \\ & 1 \leq j \leq \lceil \frac{n}{2} \rceil \\ 3, & \text{untuk } v = x_{i,2j}; 1 \leq i \leq m, \\ & 1 \leq j \leq \lceil \frac{n-1}{2} \rceil \end{cases}$$

From the function can determine $rvc(F_{m,n} \trianglerighteq Bt_p) \leq 3$. Therefore $rvc(F_{m,n} \trianglerighteq Bt_p) \geq 3$ and $rvc(F_{m,n} \trianglerighteq Bt_p) \leq 3$, then $rvc(F_{m,n} \trianglerighteq Bt_p) = 3$. □

♢ **Theorem 5.** *Let G be a comb product denote by $P_n \trianglerighteq S_m$, for $n \geq 3$ and $m \geq 3$. The rainbow vertex connection number $rvc(P_n \trianglerighteq S_m) = n - 2$.*

Proof. Let G be a comb product denote by $P_n \trianglerighteq S_m$. The vertex set of G is $V(P_n \trianglerighteq S_m) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 2 \leq i \leq n, 1 \leq j \leq m-1\}$ and the edge set is $E(P_n \trianglerighteq S_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i x_{i,j}; 2 \leq i \leq n, 1 \leq j \leq m-1\}$. The order of the graph $|V(P_n \trianglerighteq S_m)| = m(n-1) + 1$ and the size is $|E(P_n \trianglerighteq S_m)| = m(n-1)$. According Theorem 2 the lower bound are stated as follow: $rvc(P_n \trianglerighteq S_m) \geq diam(P_n \trianglerighteq S_m) - 1$, where $diam(P_n \trianglerighteq S_m)$ is the diameter of graph G. Since $diam(P_n \trianglerighteq S_m) = n - 1$, for $n \geq 3$ and $m \geq 3$, it follows that $rvc(P_n \trianglerighteq S_m) \geq n - 1 - 1$. And than, will be proof that $rvc(P_n \trianglerighteq S_m) \leq n - 2$ by construct $c : V(G) \rightarrow \{1, 2, 3, \ldots, k\}$ as follows:

$$c(v) = \begin{cases} 1, & \text{for } x_{i,j}; 2 \leq i \leq n, 1 \leq j \leq m-1 \\ 1, & \text{for } v = x_1 \\ 2, & \text{for } v = x_{i,}; 2 \leq i \leq n, \end{cases}$$

From the function can determine $rvc(P_n \trianglerighteq S_m) \leq n - 2$. Therefore $rvc(P_n \trianglerighteq S_m) \geq n - 2$ and $rvc(P_n \trianglerighteq S_m) \leq n - 2$, then $rvc(P_n \trianglerighteq S_m) = n - 2$. □

♢ **Theorem 6.** *Let G be a comb product denote by $C_n \trianglerighteq S_m$, for $n \geq 3$ and $m \geq 3$. The rainbow vertex connection number $rvc(C_n \trianglerighteq S_m) = n$.*

Proof. Let G be an exponential graph denote by $C_n \trianglerighteq S_m$. The vertex set of G is $V(C_n \trianglerighteq S_m) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$ and the edge set is $E(C_n \trianglerighteq S_m) = \{x_1 x_n, x_i x_{i+1}; 1 \leq i \leq n-2\} \cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$. The order of the graph $|V(C_n \trianglerighteq S_m)| = mn$ and the size is $|E(C_n \trianglerighteq S_m)| = n + n(m-1)$. According Theorem 2 the lower bound are stated as follow: $rvc(C_n \trianglerighteq S_m) \geq diam(C_n \trianglerighteq S_m) - 1$, where $diam(C_n \trianglerighteq S_m)$ is the diameter of graph G. Since $diam(C_n \trianglerighteq S_m) = \lfloor \frac{n}{2} \rfloor + 2$, for $n \geq 3$ and $m \geq 3$, it follows that $rvc(C_n \trianglerighteq S_m) \geq \lfloor \frac{n}{2} \rfloor + 2 - 1$. And than, will be proof that $rvc(C_n \trianglerighteq S_m) \leq \lfloor \frac{n}{2} \rfloor + 1$ by construct $c : V(G) \rightarrow \{1, 2, 3, \ldots, k\}$ as follows:

$$c(v) = \begin{cases} 1, & \text{for } x_{i,j}; 2 \leq i \leq n, 1 \leq j \leq m-1 \\ 1, & \text{for } v = x_i; 1 \leq i \leq n \end{cases}$$

From the function can determine that $rvc(C_n \trianglerighteq S_m) = n$. □

♢ **Theorem 7.** *Let G be a comb product denote by W $_n^{S_m}$, for $n \geq 4$ and $m \geq 3$. The rainbow vertex connection number $rvc(W_n \trianglerighteq S_m) = n$.*

Proof. Let G be an exponential graph denote by $W_n \trianglerighteq S_m$. The vertex set of G is $V(W_n \trianglerighteq S_m) = \{A, x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{A_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$ and the edge set is $E(W_n \trianglerighteq S_m) = \{x_n x_1, x_i x_{i+1}; 1 \leq i \leq n-2\} \cup \{A x_i; 1 \leq i \leq n\} \cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_i A_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$. The order of the graph $|V(W_n \trianglerighteq S_m)| = 2mn - n + 1$ and the size is $|E(W_n \trianglerighteq S_m)| = 2mn$. According Theorem 2 the lower bound are stated as follow: $rvc(W_n \trianglerighteq S_m) \geq diam(W_n \trianglerighteq S_m) - 1$, where $diam(W_n \trianglerighteq S_m)$ is the diameter of graph G. Since $diam(W_n \trianglerighteq S_m) = 4$, for $n \geq 4$ and $m \geq 3$, it follows that $rvc(W_n \trianglerighteq S_m) \geq 4 - 1$. And than, will be proof that $rvc(W_n \trianglerighteq S_m) \leq 3$ by

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construct $c : V(G) \to \{1, 2, 3, \ldots, k\}$ as follows:

$$c(v) = \begin{cases} 1, & \text{for } v = x_{i,j};\ 2 \leq i \leq n,\ 1 \leq j \leq m-1 \\ 1, & \text{for } v = A_{i,j};\ 2 \leq i \leq n,\ 1 \leq j \leq m-1 \\ 1, & \text{for } v = A \\ i, & \text{for } v = x_{i,};\ 1 \leq i \leq n, \end{cases}$$

From the function can determine $rvc(W_n \trianglerighteq S_m) = n$. □

Conjecture 1. *Let G be any graph. Let Bt_n be a Triangular Book graph. The rainbow vertex connection number of $G \trianglerighteq Bt_n$ is $rvc(G \trianglerighteq Bt_n) = diam(G) + diam(Bt_n) - 3$.*

Conjecture 2. *Let G be any graph. Let S_n be a Star graph. The rainbow vertex connection number of $G \trianglerighteq S_n$ is $rvc(G \trianglerighteq S_n) = n$.*

CONCLUSIONS

We have studied the *rainbow vertex connection number* of some comb product, namely $P_n \trianglerighteq Bt_m$, $S_n \trianglerighteq Bt_m$, $L_n \trianglerighteq Bt_m$, $F_{m,n} \trianglerighteq Bt_p$, and $W_n \trianglerighteq Bt_m$. The result obtain that all values of $rvc(G)$ take a place in the lower bound $rvc(G) \geq diam(G) - 1$. The result shows that for $rvc(P_n \trianglerighteq Bt_m)$, $rvc(S_n \trianglerighteq Bt_m)$, $rvc(L_n \trianglerighteq Bt_m)$, $rvc(F_{m,n} \trianglerighteq Bt_p)$, $rvc(P_n \trianglerighteq S_m)$, $rvc(C_n \trianglerighteq S_m)$, and $rvc(W_n \trianglerighteq S_m)$ are respectively as follows:

1. a comb product denote by $P_n \trianglerighteq Bt_m$ for $n \geq 2$ dan $m \geq 2$, then *rainbow vertex connection number*

$$rvc(P_n^{Bt_m}) = \begin{cases} 1; & \text{for } n = 2 \text{ and } m \geq 2 \\ n-2; & \text{for } n \geq 3 \text{ and } m \geq 2 \end{cases}$$

2. a comb product denote by $S_n \trianglerighteq Bt_m$ for $n \geq 3$ and $m \geq 2$, then *rainbow vertex connection number* $rvc(S_n \trianglerighteq Bt_m) = 1$.
3. a comb product denote by $L_n \trianglerighteq Bt_m$, for $n \geq 3$ and $m \geq 2$, then *rainbow vertex connection number* $rvc(L_n \trianglerighteq Bt_m) = n$.
4. a comb product denote by $F_{m,n} \trianglerighteq Bt_p$, for $m \geq 3$, $n \geq 2$, and $p \geq 2$, then *rainbow vertex connection number* $rvc(F_{m,n} \trianglerighteq Bt_p) = 3$.
5. a comb product denote by $P_n \trianglerighteq S_m$ for $n \geq 3$ and $m \geq 2$, then *rainbow vertex connection number* $rvc(P_n \trianglerighteq S_m) = n - 2$.
6. a comb product denote by $C_n \trianglerighteq S_m$ for $n \geq 3$ and $m \geq 2$, then *rainbow vertex connection number* $rvc(C_n \trianglerighteq S_m) = n$.
7. a comb product denote by $W_n \trianglerighteq S_m$ for $n \geq 4$ and $m \geq 2$, then *rainbow vertex connection number* $rvc(W_n \trianglerighteq S_m) = n$.

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