The Analysis of r-dynamic Vertex Colouring on Graph Operation Of Shackle

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Abstract—Let G be a simple, connected and undirected graph and r, k be natural numbers. An edge coloring that uses k colors is a k-edge coloring. Thus a graph G can be described as a function $c : V(G) \to S$, where $|S| = k$, such that any two adjacent vertices receive different colors. An r-dynamic k-coloring is a proper k-coloring c of G such that $|c(N(v))| \geq min\{r, d(v)\}$ for each vertex v in $V(G)$, where $N(v)$ is the neighborhood of v and $c(S) = \{c(v) : v \in S\}$ for a vertex subset S. The r-dynamic chromatic number, written as $\chi_r(G)$, is the minimum k such that G has an r-dynamic k-coloring. In this paper, we will study the existence of r-dynamic k-coloring when G is shackle of wheel graph. As we know, that a shackle operation of H denoted by $shack(H, v, n)$ is a shackle with vertex as the connector. We also can generated shackle graph with edge connector or subgraph as the connector.

Keywords—r-dynamic chromatic number, graph coloring, shackle graph

INTRODUCTION

According to Chartrand, an edge coloring of a graph G is an assignment of colors to the edges of G, one color to each edge. If adjacent edges are assigned distinct colors, then the edge coloring is a proper edge coloring. An r-dynamic proper k-coloring of a graph G is a proper coloring c from $V(G)$ to a set S of k colors such that $|c(N(v))| \geq min\{r, d(v)\}$ for each vertex v in $V(G)$, where $cS = \{c(v) : v \in S\}$ for a vertex subset S. The r-dynamic chromatic number of a graph G, written $\chi_r(G)$, is the minimum k such that G has an r-dynamic proper k-coloring. The dynamic chromatic number, $\chi(G)$, have been investigated in several papers, see, e.g.,[1],[2],[3],[4],[5] [6], [7],[8]for some references.

The following observation is useful to find the exact values of r-dynamic chromatic number.

Observation 1. *Let $\delta(G)$ and $\Delta(G)$ be a minimum and maximum degree of a graph G, respectively. Then the followings hold*

- $\chi_r(G) \geq \min\{\Delta(G), r\} + 1$,
- $\chi(G) \leq \chi_2(G) \leq \chi_3(G) \leq \cdots \leq \chi_{\Delta(G)}(G)$,
- $\chi_{r+1}(G) \geq \chi_r(G)$ *and if* $r \geq \Delta(G)$ *then* $\chi_r(G) = \chi_{\Delta(G)}(G)$.

THE RESULTS

We are ready to show our main theorems. There are three theorems found in this study. These deals r-dynamic chromatic number of Shack(W_n, v, m), Shack (W_n, e, m) and Shack $(W_n, H \subset W_n, m)$

Theorem 1. *Let $G = Shack(W_n, v, m)$ be a vertex shackle of wheel graph (W_n), the r-dynamic chromatic number for $n \geq 3$ is:*

$$\chi(Shack(W_n, v, m)) = \chi_d(Shack(W_n, v, m)) =$$

$$\begin{cases} 3, & n \text{ odd} \\ 4, & n \text{ even} \end{cases}$$

$$\chi_3(Shack(W_n, v, m)) = \begin{cases} 4, & n = 0 mod3 \\ 5, & n \text{ otherwise} \end{cases}$$

$$\chi_r(Shack(W_n, e, m)) = r + 1$$

Proof. Let G be a Shack (W_n, v, m), is a connected graph with vertex set $V(shack(W_n, v, m)) = \{x_i, 1 \leq i \leq m\} \cup \{y_{i,j}, 1 \leq i \leq m, 1 \leq j \leq n-1\}$ and edge set $E(Shack(W_n, v, m)) = \{x_i y_{i,j}, 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{y_{i,j} y_{i,j+1}, 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{y_{i,1} y_{i+1,j}, 1 \leq i \leq m, j = \lceil \frac{n-1}{2} \rceil\} \cup \{x_i y_{i+1,j}, 1 \leq i \leq m, j = \lceil \frac{n-1}{2} \rceil\}$. The order and size of shack(W_n, v, m) are $|V(Shack(W_n, v, m))| = nm + 1$ dan $|E(Shack(W_n, v, m))| = 2nm$. By Observation 1, $\chi_r(Shack(W_n, e, m)) \geq min\{\Delta(Shack(W_n, e, m)), r\}$

To find the exact value of r-dynamic chromatic number of $Shack(W_n, e, m)$, we define some cases, $\chi(Shack(W_n, e, m)), \chi_2(Shack(W_n, e, m))$, ..., $\chi_r(Shack(W_n, e, m))$.

For $r = 1$, the lower bound of the $\chi(Shack(W_n, e, m)) \geq min\{6, 1\} = 1$. And for $r = 2$, the lower bound $\chi(Shack(W_n, e, m)) \geq min\{6, 2\} = 2$. We will proof that $\chi_r(Shack(W_n, e, m)) \leq 3$ by defining a map $c_{\alpha 1} : V(Shack(W_n, v, m)) \to \{1, 2, ..., k\}$ where $n \geq 3$, by the following : $c_{\alpha 1}(x_i) = 1,\ 1 \leq i \leq m$

$$c_{\alpha 1}(y_{i,j}) = \begin{cases} 23\,23\,...,\ i \text{ odd},\ n \text{ even},\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 23\,23\,...,\ i \text{ odd},\ n \text{ odd and even},\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 2432\,2432\,...,\ i \text{ even},\ n \text{ odd},\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \end{cases}$$

It is easy to see that $c_{\alpha 1}$ gives $\chi(Shack(W_n, v, m)) \leq 3$ for n odd, but for n even, we could not avoid to have $\chi(Shack(W_n, v, m)) \leq 4$. From that coloring function of $c_{\alpha 1}$ we can say that $\chi(Shack(W_n, e, m)) \leq 3$ for n odd, because of $\chi(Shack(W_n, v, m)) \leq 3$ and $\chi(Shack(W_n, v, m)) \geq 3$ then $\chi(Shack(W_n, v, m)) = 3$. And for n even, $\chi(Shack(W_n, e, m)) \leq 4$ and because of $\chi(Shack(W_n, v, m)) \leq 4$ and $\chi(Shack(W_n, v, m)) \geq 4$ then $\chi(Shack(W_n, v, m)) = 4$. $\chi(Shack(W_n, e, m)) = 4$, for n even. And also for $\chi_d(Shack(W_n, v, m))$

$= 3$, the lower bound of the $\chi_r(Shack(W_n, e, m)) \geq min\{6, 3\} = 3$.We will proof that $\chi_r(Shack(W_n, e, m)) \leq 4$ by defining a map $c_{\alpha 2} : V(Shack(W_n, v, m)) \to \{1, 2, ..., k\}$ where $n \geq 3$, by the following : $c_{\alpha 2}(x_i) = 1,\ 1 \leq i \leq m$

$$c_{\alpha 2}(y_{i,j}) = \begin{cases} 23\ 23\ ...,\ n=3,\ i \text{ odd}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 43\ 43\ ...,\ n=3,\ i \text{ even}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 234\ ...\ 23,\ n = 0 mod3,\ i \text{ odd}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 342\ ...\ 34,\ n = 0 mod3,\ i \text{ even}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 234\ ...\ 2345,\ n = 1 mod3,\ i \text{ odd}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 3452\ ...\ 342,\ n = 1 mod3,\ i \text{ even}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ 2345\ ...,\ n = 2 mod3,\ i \text{ even}, \\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \end{cases}$$

It is easy to see that $c_{\alpha 2}$ gives $\chi_3(Shack(W_n, v, m)) \leq 4$ for $n = 0 mod3$, but for $n\ otherwise$, we could not avoid to have $\chi_3(Shack(W_n, v, m)) \leq 5$. From that coloring function of $c_{\alpha 3}$ we can say that $\chi_3(Shack(W_n, e, m)) \leq 4$ for $n = 0 mod3$, because of $\chi_3(Shack(W_n, v, m)) \leq 3$ and $\chi_3(Shack(W_n, v, m)) \geq 4$ then $\chi(Shack(W_n, v, m)) = 4$. And for $n\ otherwise$, $\chi_3(Shack(W_n, e, m)) \leq 5$ and because of $\chi_3(Shack(W_n, v, m)) \leq 5$ and $\chi_3(Shack(W_n, v, m)) \geq 5$ then $\chi_3(Shack(W_n, v, m)) = 5$.

For $r \geq 4$, the lower bound of the $\chi_r(Shack(W_n, e, m)) \geq min\{n, r\} = n$.We will proof that $\chi_r(Shack(W_n, e, m)) \leq n+1$ by defining a map $c_{\alpha 3} : V(Shack(W_n, v, m)) \rightarrow \{1, 2, ..., k\}$ where $n \geq 3$, by the following :

$$c_{\alpha 3}(x_i) = \begin{cases} 1,\ \ i = 1\ mod\ 3 \\ \lfloor \frac{n-2}{2} \rfloor,\ \ i = 2\ mod\ 3 \\ n+1,\ \ i = 0\ mod\ 3 \end{cases}$$

$$c_{\alpha 3}(y_{i,j}) = \begin{cases} j+1,\ \ 1 \leq j \leq \lfloor \frac{n-2}{2} \rfloor \\ and\ \lfloor \frac{n}{2} \rfloor \leq j \leq n \\ \lfloor \frac{n-2}{2} \rfloor + 2,\ \ j = \lfloor \frac{n}{2} \rfloor \\ n+1,\ \ j = n \end{cases}$$

It is easy to see that $c_{\alpha 2}$ gives $\chi_r(Shack(W_n, v, m)) \leq n+1$. From that coloring function of $c_{\alpha 3}$ we can say that $\chi_r(Shack(W_n, e, m)) \leq n+1$, because of $\chi_r(Shack(W_n, v, m)) \leq n+1$ and $\chi_r(Shack(W_n, v, m)) \geq n+1$ then $\chi_r(Shack(W_n, v, m)) = n+1$.

Theorem 2. *Let $G = Shack(W_n, e, m)$ be an edge shackle of wheel graph (W_n), the r-dynamic chromatic number for $n \geq 4$ is:*

$$\chi(Shack(W_n, e, m)) = \chi_d(Shack(W_n, e, m)) = \begin{cases} 3,\ n \text{ even} \\ 4,\ n \text{ odd} \end{cases}$$

$$\chi_r(Shack(W_n, e, m)) = r+1$$

Proof. Let G be a Shack (W_n, e, m), is a connected graph with vertex set $V(shack(W_n, v, m)) = \{x_i, 1 \leq i \leq m+1\} \cup \{y_{i,j}, 1 \leq i \leq m, 1 \leq j \leq n-4\} \cup \{z_i, 1 \leq i \leq 2m+1\}$ and edge set $E(Shack(W_n, v, m)) = \{x_i x_{i+1}, 1 \leq i \leq m\}$. The order and size of shack(W_n, v, m) are $|V(Shack(W_n, e, m))| = nm - m + 2$ dan $|E(Shack(W_n, e, m))| = 2nm - m + 1)$. By Observation 1, $\chi_r(Shack(W_n, e, m)) \geq min\{\Delta(Shack(W_n, e, m)), r\}$

To find the exact value of r-dynamic chromatic number of $Shack(W_n, v, m)$, we define some cases, namely $\chi(Shack(W_n, v, m)), \chi_2(Shack(W_n, v, m))$, ..., $\chi_r(Shack(W_n, v, m))$. For $r = 1$, the lower bound of the $\chi(Shack(W_n, v, m)) \geq min\{6, 1\} = 1$. And for $r = 2$, the lower bound $\chi(Shack(W_n, e, m)) \geq min\{6, 2\} = 2$. We will proof that $\chi(Shack(W_n, e, m)) \leq 3$ by defining a map $c_{\beta 1} : V(Shack(W_n, v, m)) \rightarrow \{1, 2, ..., k\}$ where $n \geq 3$, by the following : $c_{\beta 1}(x_i) = 23\ ...\ 23,\ 1 \leq i \leq m$ $c_{\beta 1}(z_i) = 3121\ ...\ 3121,\ 1 \leq i \leq m$

$$c_{\alpha 1}(y_{i,j}) = \begin{cases} 2\ ,\ \ j \text{ odd},\ \ n \text{ odd}, \\ 1 \leq i \leq m,\ 1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor \\ 2\ ,\ \ j \text{ even},\ \ n \text{ odd}, \\ 1 \leq i \leq m,\ \lceil \frac{n-4}{2} \rceil \leq j \leq n-4 \\ 2\ ,\ \ j \text{ even},\ \ n \text{ even}, \\ 1 \leq i \leq m,\ 1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor \\ 2\ ,\ \ j \text{ odd},\ \ n \text{ even}, \\ 1 \leq i \leq m,\ \lceil \frac{n-4}{2} \rceil \leq j \leq n-4 \\ \\ 3\ ,\ \ j \text{ even},\ \ n \text{ odd}, \\ 1 \leq i \leq m,\ 1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor \\ 3\ ,\ \ j \text{ odd},\ \ n \text{ odd}, \\ 1 \leq i \leq m,\ \lceil \frac{n-4}{2} \rceil \leq j \leq n-4 \\ 3\ ,\ \ j \text{ odd},\ \ n \text{ even}, \\ 1 \leq i \leq m,\ 1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor \\ 3\ ,\ \ j \text{ even},\ \ n \text{ even}, \\ 1 \leq i \leq m,\ \lceil \frac{n-4}{2} \rceil \leq j \leq n-4 \\ \\ 4\ ,\ \ 1 \leq i \leq m,\ j = \lceil \frac{n-4}{2} \rceil \end{cases}$$

It is easy to see that $c_{\beta 1}$ gives $\chi(Shack(W_n, e, m)) \leq 3$ for n even, but for n odd, we could not avoid to have $\chi(Shack(W_n, e, m)) \leq 4$. From that coloring function of $c_{\beta 1}$ we can say that $\chi(Shack(W_n, e, m)) \leq 3$ for n even, because of $\chi(Shack(W_n, e, m)) \leq 3$ and $\chi(Shack(W_n, e, m)) \geq 3$ then $\chi(Shack(W_n, e, m)) = 3$. And for n odd, $\chi(Shack(W_n, e, m)) \leq 4$ and because of $\chi(Shack(W_n, e, m)) \leq 4$ and $\chi(Shack(W_n, e, m)) \geq 4$ then $\chi(Shack(W_n, e, m)) = 4$. $\chi(Shack(W_n, e, m)) = 4$, for n even. And also for $\chi_d(Shack(W_n, e, m))$

For $r \geq 3$, the lower bound of the $\chi_r(Shack(W_n, e, m)) \geq min\{n, r\} = n$.We will proof that $\chi_r(Shack(W_n, e, m)) \leq n+1$ by defining a map $c_{\beta 2} : V(Shack(W_n, e, m)) \rightarrow \{1, 2, ..., k\}$ where $n \geq 3$, by the following : $c_{\beta 2}(y_{i,j}) = j+5,\ 1 \leq i \leq n-4$ $c_{\beta 2}(x_i) = 426\ ...\ 426,\ 1 \leq i \leq m+1$

$$c_{\beta 2}(z_i) = \begin{cases} 1,\ n = \ 0\ mod\ 3 \\ 3,\ n = \ 1\ mod\ 3 \\ 5,\ n = \ 0\ mod\ 3 \end{cases}$$

It is easy to see that $c_{\beta 2}$ gives $\chi_r(Shack(W_n, e, m))$ $\leq j+5$, for $1 \leq i \leq n-4$. So when j=n-4, we have $\chi_r(Shack(W_n, e, m)) \leq n+1$ From that coloring function of $c_{\beta 2}$ we can say that $\chi_r(Shack(W_n, e, m)) \leq n+1$ for, because of $\chi_r(Shack(W_n, v, m)) \leq n+1$ and $\chi_r(Shack(W_n, v, m)) \geq n+1$ then $\chi_r(Shack(W_n, v, m)) = n+1$.

Theorem 3. *Let $G = Shack(W_n, H \subset W_n, m)$ be a shackle subgraph of wheel graph (W_n), the r-dynamic chromatic number for $n \geq 6$ is:*

$$\chi(Shack(W_n, H \subset W_n, m)) =$$

$$\chi_d(Shack(W_n, H \subset W_n, m) =$$

$$\begin{cases} 3,\ n \text{ even} \\ 4,\ n \text{ odd} \end{cases}$$

$$\chi_3(Shack((W_n, H \subset W_n, m)) = \begin{cases} 4,\ n = 6 \\ 5,\ n \text{ otherwise} \end{cases}$$

$$\chi_r(Shack(W_n, H \subset W_n, m)) = r+1\ for\ n \geq r$$

Proof. Let G be a Shack $(W_n, H \subset W_n, m)$, is a connected graph with vertex set $V(shack(W_n, v, m)) =$

$\{z_i, 1 \leq i \leq 2m\} \cup \{x_{i,j}, 1 \leq i \leq m, 1 \leq j \leq \lfloor \frac{n-2}{2} \rfloor\} \cup \{y_{i,j}, 1 \leq i \leq m, 1 \leq j \leq \lceil \frac{n-2}{2} \rceil\}$ and edge set $E(Shack(W_n, H \subset W_n, m)) = \{z_1 y_{1,1}\} \cup \{z_1 x_{1,1}\} \cup \{x_{i,j} x_{i,j+1}, 1 \leq i \leq m1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor\} \cup \{x_{i,j} x_{i+1,1}, 1 \leq i \leq m1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor\} \cup \{y_{i,j} y_{i,j+1}, 1 \leq i \leq m1 \leq j \leq \lfloor \frac{n-4}{2} \rfloor + 1\} \cup \{y_{i,1} z_{i+1}, 1 \leq i \leq m\} \cup \{y_{i,1} z_i, 2 \leq i\, 2n-2\} \cup \{x_{i,1} z_{i+1}, 2 \leq i\, m\} \cup \{x_{i,1} z_i, 2 \leq i\, 2n-2\}$. The order and size of shack$(W_n, H \subset W_n, m)$ are $|V(Shack(W_n, H \subset W_n, m))| = nm - 3m + 5$ dan $|E(Shack(W_n, e, m))| = 2nm - 5m + 5)$. By Observation 1, $\chi_r(Shack(W_n, H \subset W_n, m)) \geq min\{\Delta(Shack(W_n, H \subset W_n, m)), r\}$

To find the exact value of r-dynamic chromatic number of $Shack(W_n, v, m)$, we define some cases, namely $\chi(Shack(W_n, H \subset W_n, m)), \chi_2(Shack(W_n, H \subset W_n, m))$, ..., $\chi_r(Shack(W_n, H \subset W_n, m))$. For $r = 1$, the lower bound of the $\chi(Shack(W_n, H \subset W_n, m)) \geq min\{6, 1\} = 1$. And for $r = 2$, the lower bound $\chi(Shack(W_n, H \subset W_n, m)) \geq min\{6, 2\} = 2$. We will proof that $\chi(Shack(W_n, H \subset W_n, m)) \leq 3$ by defining a map $c_{\gamma 1} : V(Shack(W_n, H \subset W_n, m)) \to \{1, 2, ..., k\}$ where $n \geq 3$, by the following :

$$c_{\gamma 1}(z_i) = \begin{cases} 213 \ldots, & n = 6 \\ 2121 \ldots, & n \geq 7 \end{cases}$$

$$c_{\gamma 1}(y_{i,j}) = \begin{cases} 321 \ldots, & n = 6 \\ 3231 \ldots, & n \text{ even} \\ 3434 \ldots, & n \text{ odd} \end{cases}$$

$$c_{\gamma 1}(x_{i,j}) = \begin{cases} 321 \ldots, & n = 6 \\ 3231 \ldots, & n \text{ even} \\ 3434 \ldots, & n \text{ odd} \end{cases}$$

It is easy to see that $c_{\gamma 1}$ gives $\chi(Shack(W_n, H \subset W_n, m)) \leq 3$ for $n = odd$, but for $neven$, we could not avoid to have $\chi_3(Shack(W_n, H \subset W_n, m)) \leq 4$. From that coloring function of $c_{\gamma 1}$ we can say that $\chi(Shack(W_n, H \subset W_n, m)) \leq 3$ for $n = odd$, because of $\chi_3(Shack(W_n, H \subset W_n, m)) \leq 3$ and $\chi_3(Shack(W_n, H \subset W_n, m)) \geq 3$ then $\chi(Shack(W_n, H \subset W_n, m)) = 3$. And for n $even$, $\chi(Shack(W_n, H \subset W_n, m)) \leq 4$ and because of $\chi(Shack(W_n, H \subset W_n, m)) \leq 4$ and $\chi(Shack(W_n, H \subset W_n, m)) \geq 4$ then $\chi(Shack(W_n, H \subset W_n, m)) = 4$. So is the $\chi_2(Shack(W_n, H \subset W_n, m))$

For $r = 3$, the lower bound of the $\chi(Shack(W_n, H \subset W_n, m)) \geq min\{6, 3\} = 3$. We will proof that $\chi(Shack(W_n, H \subset W_n, m)) \leq 4$ by defining a map $c_{\gamma 2} : V(Shack(W_n, H \subset W_n, m)) \to \{1, 2, ..., k\}$ where $n \geq 3$, by the following : $c_{\gamma 2}(z_i) = 2121 \ldots, \quad 1 \leq i \leq 2m - 1$

$$c_{\gamma 2}(y_{i,j}) = \begin{cases} 3434 \ldots, & n = 6 \\ 345345 \ldots, & n \geq 6 \end{cases}$$

$$c_{\gamma 2}(x_{i,j}) = \begin{cases} 4343 \ldots, & n = 6 \\ 345345 \ldots, & n \geq 6 \end{cases}$$

It is easy to see that $c_{\gamma 2}$ gives $\chi(Shack(W_n, H \subset W_n, m)) \leq 4$ for $n = 6$, but for $n \geq 6$, we could not avoid to have $\chi_3(Shack(W_n, H \subset W_n, m)) \leq 5$. From that coloring function of $c_{\gamma 2}$ we can say that $\chi(Shack(W_n, H \subset W_n, m)) \leq 4$ for $n = 6$, because of $\chi_3(Shack(W_n, H \subset W_n, m)) \leq 4$ and $\chi_3(Shack(W_n, H \subset W_n, m)) \geq 4$ then $\chi(Shack(W_n, H \subset W_n, m)) = 4$. And for $n \geq 6$, $\chi(Shack(W_n, H \subset W_n, m)) \leq 5$ and because of $\chi(Shack(W_n, H \subset W_n, m)) \leq 5$ and $\chi(Shack(W_n, H \subset W_n, m)) \geq 5$ then $\chi(Shack(W_n, H \subset W_n, m)) = 5$.

For $r \geq 4$, the lower bound of the $\chi(Shack(W_n, H \subset W_n, m)) \geq min\{n, r\} = n$. We will proof that $\chi(Shack(W_n, H \subset W_n, m)) \leq n + 1$ by defining a map $c_{\gamma 3} : V(Shack(W_n, H \subset W_n, m)) \to \{1, 2, ..., k\}$ where $n \geq 3$, by the following :
$c_{\gamma 3}(z_i) = j + 3$, $1 \leq j \leq \lfloor \frac{n-2}{2} \rfloor$
$c_{\gamma 3}(z_i) = j + 3 + \lceil \frac{n-2}{2} \rceil$, $1 \leq j \leq \lfloor \frac{n-2}{2} \rfloor$

$$c_{\beta 2}(z_i) = \begin{cases} 3, & n = 0 \bmod 3 \\ 2, & n = 1 \bmod 3 \\ 1, & n = 0 \bmod 3 \end{cases}$$

It is easy to see that $c_{\gamma 3}$ gives $\chi_r(Shack(W_n, H \subset W_n, m)) \leq j + 3 + \lceil \frac{n-2}{2} \rceil$ then we have $j = \lfloor \frac{n-2}{2} \rfloor$ so $\chi_r(Shack(W_n, H \subset W_n, m)) \leq n + 1$. From that coloring function of $c_{\gamma 2}$ we can say that $\chi_r(Shack(W_n, H \subset W_n, m)) \leq n + 1$ for $n \geq 6$, because of $\chi_r(Shack(W_n, H \subset W_n, m)) \leq n + 1$ and $\chi_3(Shack(W_n, H \subset W_n, m)) \geq n + 1$ then $\chi(Shack(W_n, H \subset W_n, m)) = n + 1$.

CONCLUSIONS

We have found the $r-$dynamic chromatic number of $Shack(W_n, v, m), Shack(W_n, e, m)$ and $Shack(W_n, H \subset W_n, m)$. It is interesting to characterize a property of any graph operation of shackle to have an exact value or upper bound of their $r-$dynamic chromatic numbers.

Conjecture 1. *Let G=Shack* $(W_n, H \subset W_n, m)$ *then upper bound of vertex r-dynamic chromatic number is* $\chi_r(Shack(W_n, v, m)) = r + 1$ *for* $n \geq r$

REFERENCES

[1] M. Alishahi, "Dynamic chromatic number of regular graphs", Discrete Appl. Math., vol. 160, pp. 2098-2103, 2012.

[2] M. Alishahi, "On the dynamic coloring of graphs", Discrete Appl. Math., vol. 159, pp. 152-156, 2011.

[3] S. Akbari, M. Ghanbari and S. Jahanbekam, "On the list dynamic coloring of graphs", Discrete Appl. Math., vol. 157, pp. 3005-3007, 2009.

[4] S. Akbari, M. Ghanbari and S. Jahanbekam, "On the dynamic chromatic number of graphs", in: Combinatorics and Graphs, in: Contemp. Math., Amer. Math. Soc., vol. 531, pp. 1-18, 2010.

[5] S. Jahanbekama, J. Kimb, O. Suil and D. B. Westd, "On r-dynamic Coloring of Graphs", Discrete Applied Mathematics, 2016, In Press.

[6] B. Montgomery, "Dynamic Coloring of Graphs", Ph.D Dissertation, West Virginia University, 2001.

[7] H.J. Lai, B. Montgomery and H. Poon, "Upper bounds of dynamic chromatic number", Ars Combin., vol. 68, pp. 193–201, 2003.

[8] A. Taherkhani, "r-Dynamic Chromatic Number of Graphs", arXiv:1401.6470v1 [math.CO], Iran: Dept of Math, Institutu for Basic Studies and Advance Sciense, 2014.