On The Metric Dimension with Non-isolated Resolving Number of Some Exponential Graph

S.M. Yunika[1,2], Slamin[1,4], Dafik[1,3], Kusbudiono[1,2]
[1]CGANT University of Jember Indonesia [2]Mathematics Depart. University of Jember Indonesia [3]Mathematics Edu. Depart. University of Jember Indonesia
[4]System Information. Depart. University of Jember
e-mail: nichachapri@gmail.com

Abstract—Let $w, w \in G = (V, E)$. A distance in a simple, undirected and connected graph G, denoted by $d(v, w)$, is the length of the shortest path between v and w in G. For an ordered set $W = \{w_1, w_2, w_3, \dots, w_k\}$ of vertices and a vertex $v \in G$, the ordered k-vector $r(v|W) = (d(v, w_1), d(v, w_2), \dots, d(v, w_k))$ is representations of v with respect to W. The set W is called a resolving set for G if distinct vertices of G have distinct representations with respect to W. The metric dimension $dim(G)$ of G is the minimum cardinality of resolving set for G. The resolving set W of graph G is called non-isolated resolving set if subgraph W is induced by non-isolated vertex. While the minimum cardinality of non-isolated resolving set in graph is called a non-isolated resolving number, denoted by $nr(G)$. In this paper we study a metric dimension with non-isolated resolving number of some exponential graph.

Keywords—Metric dimension, Non-isolated resolving number, Exponential graph.

INTRODUCTION

A graph G is pair set (V, E) where V is not empty set of element called vertex, and E a set unordered pair of two vertices $(v1, v2)$ where $v1, v2 \in V$, is called edges[1]. V is called vertex set o G, and E is called edge set of G[2]. In the year of 1975, concept metric dimension introduced by Slater [3]. This concept show of resolving set known as locating set. Resolving set W defined as resolving of vertices in graph G so that for every vertices in G produce distance different of every vertex in W. Dimension metric is cardinality minimum of resolving set of graph G which denoted by $dim(G)$ [4]. For sequence resolving $W = w_1, w_2, w_3, ..., w_n$ of vertex set in graph connected G and vertex v in G, k-vector (k-tuple sequences) $r = (v|W) = (d(v, w_1), d(v, w_2), ..., d(v, w_k))$[5]. Resolving set W of graph G is called non-isolated resolving set if subgraph W induced by isolated vertices. Cardinality minimum of non-isolated resolving number denoted by $nr(G)$[6]. Related research metric dimension is [7], [8], [9], [10].In this research resulted metric dimension with non-isolated resolving number of some exponential graph.

THE RESULTS

We are ready to show our main theorems. There are two theorems found in this study. These deals with exponential graph $L_n^{P_m}$ and $S_n^{P_m}$

Theorem 1. *For $n \geq 3$ and $m \geq 4$, the metric dimension and non-isolated resolving number of $L_n^{P_m}$ are $dim(L_n^{P_m}) = 2$ and $nr(L_n^{P_m}) = m$.*

Proof. A exponential graph of $L_n^{P_m}$ is a connected graph with vertex set $V(L_n^{P_m}) = \{x_i, y_i, z_{i,j};\ 1 \leq i \leq n\ 1 \leq j \leq m-2\} \cup \{x_{i,j}, y_{i,j};\ 1 \leq i \leq n-1;\ 1 \leq j \leq m-2\}$, and edge set $E(L_n^{P_m}) = \{x_i x_{i,1}, y_i y_{i,1};\ 1 \leq i \leq n-1\} \cup \{y_i z_{i,1};\ 1 \leq i \leq n-1; 1 \leq i \leq n-1\} \cup \{x_{i,j} x_{i,j+1}, y_{i,j} y_{i,j+1};\ 1 \leq i \leq n-1; 1 \leq j \leq m-3\} \cup \{z_{i,j} z_{i,j+1};\ 1 \leq i \leq n; 1 \leq j \leq m-3\} \cup \{y_{i+1} y_{i,m-2}, x_{i+1} x_{i,m-2};\ 1 \leq i \leq n-1\} \cup \{x_i z_{i,m-2};\ 1 \leq i \leq n\}$. Thus $|V(L_n^{P_m})| = 3nm - 2m - 4n + 4$ and $|E(L_n^{P_m})| = 3nm - 2m - 3n + 2$.

For $n \geq 3$ and $m \geq 4$, the minimum cardinality of resolving set is two, If it is one then there will be the same representation of every $v \in V(L_n^{P_m})$ with respect to W are as follow:

$r(x_i|W) = (mi-i, mi-m-i+1); 1 \leq i \leq n$
$r(y_i|W) = (mi-m-i+1, mi-i); 1 \leq i \leq n$
$r(x_{i,j}|W) = (m+3i+j-4, 3i+j-3); 1 \leq i \leq n-1; 1 \leq j \leq m-2$
$r(y_{i,j}|W) = \{(3i+j-3, m+3i+j-4);\ 1 \leq i \leq n-1;\ 1 \leq j \leq m-2\}$
$r(z_{i,j}|W) = \{(3i+j-3, 3i-j);\ 1 \leq i \leq n;\ 1 \leq j \leq m-2\}$

It is easy to see that $L_n^{P_m}$ has a different representation of every $v \in V(L_n^{P_m})$ with respect to W. □
Therefore) $\mathbf{dim}(L_n^{P_m})$ It concludes that $\mathbf{dim}(L_n^{P_m}) = 2$

The next it will be showed by $nrL_n^{P_m}$. Based on observations Arumugam which state that $nr(L_n^{P_m}) \geq dim(L_n^{P_m})$.So That $nr(L_n^{P_m}) \geq dim(L_n^{P_m}) = 2$, however $nr(L_n^{P_m}) \neq 2$ because it does not fulfill the character *non-isolated resolving set* that $W = \{x_1, y_1\}$ it consists of dots which is not connected each other. For $n \geq 3$ and $m \geq 4$, the minimum cardinality of non-isolated resolving set is m, otherwise there will be the same representation of every $v \in V(L_n^{P_m})$ with respect to W'. Thus the lower bound of $\mathbf{nr}(L_n^{P_m}) \geq m$. We will show that $\mathbf{nr}(L_n^{P_m}) \leq m$, by choosing $W' = \{x_1, y_1, z_{1,j}; 1 \leq j \leq m-2\}$ as a non-isolated resolving set. Clearly that cardinality of $|W'| = m$. The representation of every $v \in V(L_n^{P_m})$ with respect to W' are as follow:

$r(x_i|W) = \{(a_{i,k}); a_{i,k} = mi-i-k+1; 1 \leq i \leq n; 1 \leq k \leq m\}$
$r(y_i|W) = \{(b_{i,k}); b_{i,k} = mi-m-i-k; 1 \leq i \leq n; 1 \leq k \leq m\}$
$r(x_{i,j}|W) = \{(a_{i,j,k}); a_{i,j,k} = mi-i+j-k+1; 1 \leq i \leq n-1; 1 \leq j \leq m-2; 1 \leq k \leq m\}$
$r(y_{i,j}|W) = \{(b_{i,j,k}); b_{i,j,k} = mi-i-m+j+k; 1 \leq i \leq n-1; 1 \leq j \leq m-2; 1 \leq k \leq m\}$
$r(z_{i,j}|W) = \{(c_{i,j,k}); 1 \leq i \leq n; 1 \leq j \leq m-2; 1 \leq k \leq m\}$

where

$$c_{i,j,k} = \begin{cases} j, & \text{for } i = 1;\ 1 \le j \le m-2;\ k = 1 \\ k - j - 1, & \text{for } i = 1;\ 1 \le j \le m-2;\ j+1 \le k \le m \\ j - k + 1, & \text{for } i = 1;\ 2 \le j \le m-2;\ 2 \le k \le j \\ im - m - i + k + j, & \text{for } 2 \le i \le n;\ 1 \le j \le m-2;\ 1 \le k \le m - j \\ im + m - i - k - j, & \text{for } 2 \le i \le n;\ 1 \le j \le m-2;\ m - j \le k \le m \end{cases}$$

It is easy to see that $L_n^{P_m}$ has a different representation of every $v \in V(L_n^{P_m})$ with respect to W'. Therefore $\mathbf{nr}(L_n^{P_m}) \le m$. It concludes that $\mathbf{nr}(L_n^{P_m}) = m$ for $n \ge 3$ and $m \ge 4$. □

Theorem 2. *For $n \ge 3$ and $m \ge 4$, the metric dimension and non-isolated resolving number of $S_n^{P_m}$ are $dim(L_n^{P_m}) = n - 1$ and $nr(L_n^{P_m}) = n$.*

Proof. A exponential graph of $S_n^{P_m}$ is a connected graph with vertex set $V(S_n^{P_m}) = \{A\} \cup \{x_i;\ 1 \le i \le n\} \cup \{x_{i,j};\ 1 \le i \le n;\ 1 \le j \le m-2\}$, and edge set $E(S_n^{P_m}) = \{Ax_{i,1}, x_i xi, m-2;\ 1 \le i \le n\} \cup \{x_{i,j}x_{i,j+1};\ 1 \le i \le n; 1 \le j \le m-3\}$. Thus $|V(S_n^{P_m})| = n + nm - 2n + 1$ and $|E(S_n^{P_m})| = nm - n$.

For $n \ge 3$ and $m \ge 4$, the minimum cardinality of resolving set is $n - 1$, otherwise there will be the same representation of every $v \in V(S_n^{P_m})$ with respect to W. Thus the lower bound of $\mathbf{dim}(S_n^{P_m}) \ge n - 1$. We will show that $\mathbf{dim}(S_n^{P_m}) \le n - 1$, by choosing $W = \{x_{i,1}; 1 \le i \le n-1\}$ as a resolving set. Clearly that cardinality of $|W| = n - 1$. The representation of every $v \in V(S_n^{P_m})$ with respect to W are as follow:

$r(A|W) \quad = \{(a_k); a_k = 1; 1 \le k \le n-1\}$

$r(x_i|W) \quad = \{(a_{i,k}); 1 \le i \le n; 1 \le k \le n-1\}$

where

$$a_{i,k} = \begin{cases} m-2, & \text{for } i = k \\ m, & \text{for } i \text{ and } k \text{ other} \end{cases}$$

$r(x_{i,j}|W) \quad = \{(b_{i,j,k}); 1 \le i \le n; 1 \le j \le m-2; 1 \le k \le n-1\}$

where

$$b_{i,j,k} = \begin{cases} j-1, & \text{for } 1 \le i \le n-1;\ 1 \le j \le m-2; k = i \\ j+1, & \text{for } 1 \le i \le n;\ 1 \le j \le m-2; k \ne i \end{cases}$$

It can be seen that every vertex of graph $S_n^{P_m}$ has a different representation to W, so the cardinality minimum resolving set of graph $S_n^{P_m}$ which chosen is $|W| = |\{x_{i,1}; 1 \le i \le n-1\}| = n - 1$ or $dim(S_n^{P_m}) \le n - 1$. Therefore proved that $dim(S_n^{P_m}) = n - 1$ for $n \ge 3$ and $m \ge 4$. □

The next it will be showed by $nrS_n^{P_m}$. Based on observations Arumugam which state that $nr(S_n^{P_m}) \ge dim(S_n^{P_m})$. So That $nr(S_n^{P_m}) \ge dim(S_n^{P_m}) = n - 1$, however $nr(S_n^{P_m}) \ne n - 1$, because it does not the character non-isolated resolving set that $W = \{x_{i,1}; 1 \le i \le n-1\}$ it consist of vertex which is not connected each other. For $n \ge 3$ and $m \ge 4$, the minimum cardinality of non-isolated resolving set is n, otherwise there will be the same representation of every $v \in V(S_n^{P_m})$ with respect to W'. Thus the lower bound of $\mathbf{dim}(S_n^{P_m}) \ge n$. We will show that $\mathbf{nr}(S_n^{P_m}) \le n$, by choosing $W' = \{A, x_{i,1}; 1 \le i \le n-1\}$ as a non-isolated resolving set. Clearly that cardinality of $|W'| = n$. The representation of every $v \in V(S_n^{P_m})$ with respect to W' are as follow:

$r(A|W') \quad = \{(a_j); a_j = 1; 1 \le j \le n-1\}$

where

$$a_k = \begin{cases} 0, & \text{for } k = 1 \\ 1, & \text{for } 2 \le k \le n-1 \end{cases}$$

$r(x_i|W') \quad = \{(b_{i,k}); 1 \le i \le n; 1 \le k \le n-1\}$

where

$$b_{i,k} = \begin{cases} m-2, & \text{for } 1 \le i \le n-1\,; k = i+1 \\ m-1, & \text{for } i = k \\ m, & \text{for } i \text{ and } k \text{ other} \end{cases}$$

$r(x_{i,j}|W') \quad = \{(c_{i,j,k}); 1 \le i \le n; 1 \le j \le m-2; 1 \le k \le n-1\}$

where

$$c_{i,j,k} = \begin{cases} j, & \text{for } 1 \le i \le n; 1 \le j \le m-2; k = 1 \\ j-1, & \text{for } 1 \le i \le n-1; 1 \le j \le m-2; k = i+1 \\ j+1, & \text{for } 1 \le i \le n; 1 \le j \le m-2; k \ne 1 \text{ and } k \ne i+1 \end{cases}$$

It is easy to see that $S_n^{P_m}$ has a different representation of every $v \in V(L_n^{P_m})$ with respect to W'. Therefore $\mathbf{nr}(S_n^{P_m}) \le n$. It concludes that $\mathbf{nr}(S_n^{P_m}) = n$ for $n \ge 3$ and $m \ge 4$. □

CONCLUDING REMARKS

We have shown the metric dimension number and metric dimension non-isolated resolving number of exponential graph, namely $L_n^{P_m}$ and $S_n^{P_m}$. The results show that the metric dimension numbers and metric dimension non-isolated resolving numbers attain the best lower bound. However we have not found the sharpest lower bound for general graph, therefore we proposed the following open problem.

Open Problem 1. *Let H be any graph and P_n be a graph, obtain the best lower bound of the metric dimension number and metric dimension non-isolated resolving number of H^{P_n}.*

ACKNOWLEDGEMENT

We gratefully acknowledge the support from DP2M research grant Fundamental and CGANT - University of Jember of year 2016.

REFERENCES

[1] F. Harary and R. A. Melter, ”On the metric dimension of a graph”, Ars Combin, issue 2, pp. 191-195, 1976.

[2] N. Hartsfield and G. Ringel, ”Pearls in Graph Theory”, London: Accademic Press Limited, 1994.

[3] G. Chartrand and L. Lesniak, ”Graph and Digraph, California: Pasifik Graw, 1986.

[4] F. Harary, ”Graph Teory”, Wesley Publishing Company, Inc, 1969.

[5] C. Hernando, et al., ”On The Metric Dimension of Some Families of Graphs”. Preprint.

ISBN: 978-602-60569-5-5

[6] P. J. B. Chitra and S. Arumungan, ”Resolving Sets Without Isolted Vertices”, India: Kalasalingan University, 2000.

[7] M. Feng, et al., ”On the metric dimension of line graphs”, Original Research Article Discrete Applied Mathematics, vol. 161, issue 6, pp. 802-805, 2013.

[8] C. Grigorious, et al., ”On the Metric Dimension of Circulant and Harary Graphs”, Original Research Article Applied Mathematics and Computation, vol. 248, pp. 47-54, 2014.

[9] M. Imran, et al., ”On the Metric Dimension of Circulant Graphs”, Original Research Article Applied Mathematics Letters, vol. 25, issue 3, pp. 320-325, 2012.

[10] I. G. Yero, et al., ”On the metric dimension of corona product graphs”. Original Research Article Computers and Mathematics with Applications”, vol. 61, issue 9, pp. 2793-2798, 2011.