Construction of Super H-Antimagicness of Graph by Uses a Partition Technique with Cancelation Number

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Abstract— The graph operation is one method to construct a new graph by applying the operation to two or more graph. One of graph operation is amalgamation, let $\{H_i\}$ be a finite collection of nontrivial, simple and undirected graphs and let each H_i has a fixed vertex v_j called a terminal. The terminal of graph operation is formed by taking all the H_i's and identifying their terminal. When H_i are all isomorphic graphs, for any positif integer n, we denote such amalgamation by $G = \text{Amal}(H, v, n)$, where n denotes the number of copies of H and v is the terminal. The graph G is said to be an (a, d)-H-antimagic total graph if there exist a bijective function $f : V(G) \cup E(G) \to \{1, 2, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to H. The total H-weights $W(H) = \sum_{v\in V(H)} f(v) + \sum_{e\in E(H)} f(e)$ form an arithmetic sequence $\{a, a + d, a + 2d, ..., a + (n - 1)d\}$, where a and d are positive integers and n is the number of all subgraphs isomorphic to H. An (a, d)-H-antimagic total labeling f is called super if the smallest labels appear in the vertices. In this paper, we study a super (a, d)-H antimagic total labeling of connected of graph $G = \text{Amal}(H, P_{s+2}, n)$ by uses a partition technique with cancelation number. The result is graph $G = \text{Amal}(H, P_{s+2}, n)$ admits a super(a, d)-H antimagic total labeling for almost feasible difference d.

Keywords—Super H-antimagic total graph, Amalgamation of graph.

INTRODUCTION

Antimagicness total labelling on the graph G is a mapping of the set edge or vertex to the set of positive integers. It is a bijective function $f : V(G) \cup E(G) \to \{1, 2, \dots, |V(G)| + |E(G)|\}$. Such that for all subgraphs isomorphic to H, admit the total H-weight $w(H) = \sum_{v\in V(H)} f(v) + \sum_{e\in E(H)} f(e)$ form an arithmetic sequence $\{a, a + d, a + 2d, ..., a + (n - 1)d\}$, where a and d are positive integers and n is the number of all subgraphs of G isomorphic to H. It could be called an (a, d)-H-antimagic total labeling of G if such a function exist. An (a, d)-H-antimagic total labeling f is called super if $f : V(G) \to \{1, 2, \dots, |V(G)|\}$.

Some articles about (a, d)-H-antimagic total labeling can be cited in [**?**, **?**, 3, 4, 5, 6, 7, 8, 9]. One of article at Inayah *et al.* in [3] explaining about connected graph. They proved that, for H is a non-trivial connected graph and $k \geq 2$ is an integer, $shack(H, v, k)$ which contains exactly k subgraphs isomorphic to H admit H-super antimagic. But they only covered a connected version of shackle of graph when a vertex as a connector, and their paper did not cover all feasible d. Our paper attempt to solve a super (a, d)-H antimagic total labeling of $G = \text{Amal}(H, P_{s+2}, n)$ for connected graph when H is any graph and the terminal is path for feasible d.

To show those existence, we will use *an integer set partition technique* introduced by [10, **?**]. This technique used in determining the feasible difference d. Let n, m, and d be positive integers. We consider the partition $\mathcal{P}^n_{m,d}(i, j)$ of the set $\{1, 2, \dots, mn\}$ into n columns, $n \geq 2$, m-rows such that the difference between the sum of the numbers in the $(j + 1)$th m-rows is always equal to the constant d, where $j = 1, 2, \dots, n - 1$. Figure 1 is the illustration of graph $G = \text{Amal}(H, P_{s+2}, n)$.

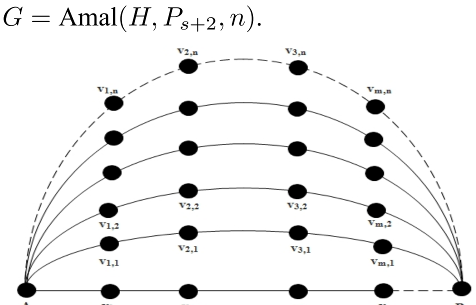

Fig 1. Illustration of graph $G = \text{Amal}(H, P_{s+2}, n)$

A USEFUL LEMMA AND COROLLARY

Let G be an amalgamation of any graph H, denoted by $G = \text{Amal}(H, P_{s+2}, n)$. The graph G is a connected graph with $|V(G)| = p_G$, $|E(G)| = q_G$, $|V(H)| = p_H$, and $|E(H)| = q_H$. The vertex set and edge set of the graph $G = \text{Amal}(H, P_{s+2}, n)$ are: $V(G) = \{A, B\} \cup \{v_k; 1 \leq k \leq s, s = \lfloor \frac{n}{2} \rfloor m - \lfloor \frac{n}{2} \rfloor\} \cup \{v_{i,j}, 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{Av_1, v_kv_{k+1}, v_sB; 1 \leq k \leq s, s = \lfloor \frac{n}{2} \rfloor m - \lfloor \frac{n}{2} \rfloor\} \cup \{e_{lj}; 1 \leq l \leq r, 1 \leq j \leq n\}$. Let n, m be positive integers with $n \geq 2$ and $m \geq 3$. Thus $|V(G)| = p_G = (s + 2) + mn$ and $|E(G)| = q_G = (s + 1) + nr$.

The upper bound of feasible d for $G = \text{Amal}(H, P_{s+2}, n)$ to be a super (a, d)-H-antimagic total labeling follows the following lemma [**?**].

Lemma 1. *[?] Let G be a simple graph of order p and size q. If G is super (a, d)-H-antimagic total labeling then* $d \leq \frac{(p_G - p_H)p_H + (q_G - q_H)q_H}{n-1}$, *for* $p_G = |V(G)|$, $q_G = |E(G)|$, $p_H = |V(H)|$, $q_H = |E(H)|$, *and* $n = |H_j|$.

Thus for $p_G = np_H - (n - 1)(s + 2)$ and $q_G = nq_H - (n - 1)(s + 1)$, we have the following corollary.

Corollary 1. *For* $n \geq 2$, *if the graph* $G = \text{Amal}(H, P_{s+2}, n)$ *admits super* (a, d)-H-*antimagic total labeling then* $d \leq p_H^2 + q_H^2 - (s + 2)p_H - (s + 1)q_H$

We will use the partition for a linear combination in developing a bijection of vertex and edge label of the main theorem.

Lemma 2. *[?] Let n and m be positive integers. The sum of* $\mathcal{P}^n_{m,d_1}(i, j) = \{(i - 1)n + j, \;\; 1 \leq i \leq m\}$ *and* $\mathcal{P}^n_{m,d_2}(i, j) = \{(j - 1)m + i; \;\; 1 \leq i \leq m\}$ *form an aritmatic sequence of difference* $d_1 = m, d_2 = m^2\}$, *respectively.*

Proof. By simple calculation, for $1 \leq j \leq n$, it gives $\sum \mathcal{P}^n_{m,m}(i, j) = \frac{m^2n - nm}{2} + mj \longleftrightarrow \sum \mathcal{P}^n_{m,m}(i, j) = \{\frac{m^2n-nm}{2} + m, \frac{m^2n-nm}{2} + 2m, \dots, \frac{m^2n-nm}{2} + (n - 1)m, \dots, \frac{m^2n-nm}{2} + nm\}$ and $\sum \mathcal{P}^n_{m,m^2}(i, j) = \frac{(m-m^2)}{2} + m^2k \longleftrightarrow \sum \mathcal{P}^n_{m,m^2}(i, j) = \{\frac{(m-m^2)}{2} + m^2, \frac{(m-m^2)}{2} + 2m^2, \dots, \frac{m^2n-nm}{2} + (n - 1)m^2, \dots, \frac{m^2n-nm}{2} + nm^2\}$. It concludes the proof. □

Lemma 3. *Let n and m be positive integers. For $1 \le j \le n$, the sum of $\mathcal{P}^n_{m,d_3}(i,j) = \{1 + ni - j; 1 \le i \le m\}$ and $\mathcal{P}^n_{m,d_4}(i,j) = \{mn + i - mj; 1 \le i \le m\}$ form an arithmetic sequence of differences $d_3 = -m, d_4 = -m^2$.*

Proof. By simple calculation, for $j = 1, 2, ..., n$, it gives $\sum_{i=1}^{m} \mathcal{P}^n_{m,-m}(i,j) = \mathcal{P}^n_{m,-m}(j) \longleftrightarrow \mathcal{P}^n_{m,-m}(j) = \{\frac{n}{2}(m^2+m) + m - mj\} \longleftrightarrow \mathcal{P}^n_{m,-m}(j) = \{\frac{n}{2}(m^2+m), \frac{n}{2}(m^2+m) - m, \frac{n}{2}(m^2+m) - 2m, \dots, \frac{n}{2}(m^2+m)m - mn\}$ and $\sum_{i=1}^{m} \mathcal{P}^n_{m,-m^2}(i,j) = \mathcal{P}^n_{m,-m^2}(j) \longleftrightarrow \mathcal{P}^n_{m,-m^2}(j) = \{\frac{m}{2}(2mn + m + 1) - m^2 j\} \longleftrightarrow \mathcal{P}^n_{m,-m^2}(j) = \{\frac{m}{2}(2mn + m + 1) - m^2, \frac{m}{2}(2mn + m + 1) - 2m^2, \dots, \frac{m}{2}(2mn + m + 1) - m^2 n\}$. It is easy to see that the differences of those sequences are $d_3 = -m, d_4 = -m^2$. It concludes the proof. □

THE RESULTS

As mentioned above, we will use a special technique, namely *an integer set partition technique*. We consider the partition $\mathcal{P}^n_{m,d}(i,j)$ of the set $\{1, 2, \dots, mn\}$ into n columns with $n \ge 2$, m-rows such that the difference between the sum of the numbers in the $(j+1)$th m-rows and the sum of the numbers in the jth m-rows is always equal to the constant d, where $j = 1, 2, \dots, n-1$. We put a cancellation number from a set partition on path. Thus these sums form an arithmetic sequence with the difference d. We need to establish some lemmas related to the partition $\mathcal{P}^n_{m,d}(i,j)$. These lemmas are useful to develop the super (a, d)-H antimagic total labeling of $G = \text{Amal}(H, P_{s+2}, n)$.

Lemma 4. *Let n and m be positive integers. For $1 \le i \le m$ and $1 \le j \le n$, the sum of*

$$\mathcal{P}^n_{m,d_5}(i,j) = \begin{cases} \frac{j-1}{2}(3m-1) + i; \\ j \text{ odd} \\ (m-1) + \frac{j-2}{2}(3m-1) + 2i; \\ j \text{ even} \end{cases}$$

form an aritmatic sequence of difference $d_5 = \frac{3m^2-m}{2}$.

Proof. By simple calculation, it gives $\sum_{i=1}^{m} \mathcal{P}^n_{m,d_5}(i,j) = \mathcal{P}^n_{m,d_5}(j) = (\frac{3m^2-m}{2})j - m^2 + m \longleftrightarrow \mathcal{P}^n_{m,d_5}(j) = \{(\frac{3m^2-m}{2}) - m^2 + m, 2m^2, \dots, (\frac{3m^2-m}{2})(n-1) - m^2 + m, (\frac{3m^2-m}{2})n - m^2 + m\}$. It concludes the proof. □

Lemma 5. *Let n and m be positive integers. For $1 \le i \le m$ and $1 \le j \le n$, the sum of*

$$\mathcal{P}^n_{m,d_6}(i,j) = \begin{cases} (3m-1)(\lfloor\frac{n+1}{2}\rfloor - \frac{1}{2}) + \frac{j}{2} \\ (3m-1) + (i-1) \\ ;j \text{ odd} \\ (3m-1)(\lceil\frac{n-1}{2}\rceil - 1) + \frac{j-2}{2} \\ (-3m+1) + 2i - m - 1 \\ ;j \text{ even} \end{cases}$$

form an aritmatic sequence of difference $d_6 = -(\frac{3m^2-m}{2})$.

Proof. By simple calculation, it gives $\sum_{i=1}^{m} \mathcal{P}^n_{m,d_6}(i,j) = \mathcal{P}^n_{m,d_6}(j) = (\frac{m^2(n+2)-m}{2} + \lfloor\frac{n+1}{2}\rfloor(\frac{3m^2-m}{2} + m) + (\frac{-3m^2+m}{2})j \longleftrightarrow \mathcal{P}^n_{m,d_6}(j) = \{(\frac{m^2(n+2)-m}{2} + \lfloor\frac{n+1}{2}\rfloor(\frac{3m^2-m}{2} + m) + (\frac{-3m^2+m}{2}), (\frac{m^2(n+2)-m}{2} + \lfloor\frac{n+1}{2}\rfloor(\frac{3m^2-m}{2} + m) - 3m^2 + m, \dots, (\frac{m^2(n+2)-m}{2} + \lfloor\frac{n+1}{2}\rfloor(\frac{3m^2-m}{2} + m) + (\frac{-3m^2+m}{2})n\}$. We have the desired difference. □

To show the existence of super $(a,d) - H$ antimagicness of the connected graph $G = \text{Amal}(H, P_{s+2}, n)$, we present our main theorem above.

Theorem 1. *For $n \ge 2$, the graph $G = \text{Amal}(H, P_{s+2}, n)$ admits a super $(a,d) - H$ antimagic total labeling with feasible d is $0 \le d \le p_H^2 + q_H^2 - (s+2)p_H - (s+1)q_H$.*

Proof. Let m and r be positive integers, with $m = p_H - (s+2)$, $r = q_H - (s+1)$ where $s = \lfloor\frac{n}{2}\rfloor m - \lfloor\frac{n}{2}\rfloor$ For $1 \le i \le m$ and $1 \le j \le n$. The vertex labeling f is a bijective function $f : V(G) \cup E(G) \to \{1, 2, \dots, p_G + q_G\}$. We can define the vertex and the edge labels by using Lemma $2, 3, 4$ and 5 as a linear combination of $\mathcal{P}^n_{m_1,m_1}(i,j)$; $\mathcal{P}^n_{m_2,-m_2}(i,j)$; $\mathcal{P}^n_{m_3,m_3^2}(i,j)$; $\mathcal{P}^n_{m_4,-m_4^2}(i,j)$;$\mathcal{P}^n_{m_5,\frac{3m_5^2-m_5}{2}}(i,j)$; and $\mathcal{P}^n_{m_6,-(\frac{3m_6^2-m_6}{2})}(i,j)$ as follows:

$$\begin{aligned}
f(v_{i,j}) &= \{\mathcal{P}^n_{m_1,m_1}(i,j)\} \cup \{\mathcal{P}^n_{m_2,-m_2}(i, j) \oplus \\
&\quad nm_1\} \cup \{\mathcal{P}^n_{m_3,m_3^2}(i,j) \oplus n(\sum_{t=1}^{2} m_t)\} \\
&\quad \cup \{\mathcal{P}^n_{m_4,-m_4^2}(i,j) \oplus n(\sum_{t=1}^{3} m_t\} \cup \\
&\quad \{\mathcal{P}^n_{m_5,\frac{-3m_5^2+m_5}{2}}(i,j) \oplus n(\sum_{t=1}^{4} m_t)\} \\
&\quad \cup \{\mathcal{P}^n_{m_6,\frac{-3m_6^2+m_6}{2}}(i,j) \oplus n(\sum_{t=1}^{5} m_t) \\
&\quad + (m_5 - 1)\lfloor\frac{n}{2}\rfloor\} \\
f(v_s) &= \{(m_5+2) + (x-1)(3m_5 - 1) \\
&\quad + 2(y-1) \oplus n(\sum_{t=1}^{4} m_t); \\
&\quad 1 \le x \le \frac{s_1}{m_5 - 1}; 1 \le y \le m_5 - 1\} \\
&\quad \cup \{(m_6+2) + (x-1)(3m_6 - 1) \\
&\quad + 2(y-1) \oplus n(\sum_{t=1}^{4} m_t); \\
&\quad 1 \le x \le \frac{s_2}{m_6 - 1}; 1 \le y \le m_6 - 1)\} \\
f(A,B) &= \{n(\sum_{t=1}^{6} m_t) + (m_5 + m_6 - 1)\lfloor\frac{n}{2}\rfloor - 1, \\
&\quad n(\sum_{t=1}^{6} m_t) + (m_5 + m_6 - 1)\lfloor\frac{n}{2}\rfloor\} \\
f(e_{l,j}) &= \{\mathcal{P}^n_{r_1,r_1}(i,j) \oplus n(\sum_{t=1}^{6} m_t) \\
&\quad + (m_5 + m_6 - 1)\lfloor\frac{n}{2}\rfloor\} \\
&\quad \cup \{\mathcal{P}^n_{r_2,-r_2}(i,j) \oplus n(\sum_{t=1}^{6} m_t) \\
&\quad + (m_5 + m_6 - 1)\lfloor\frac{n}{2}\rfloor + nr_1\} \\
&\quad \cup \{\mathcal{P}^n_{r_3,r_3^2}(i,j) \oplus n(\sum_{t=1}^{6} m_t) \\
&\quad + (m_5 + m_6 - 1)\lfloor\frac{n}{2}\rfloor + n(\sum_{t=1}^{2} r_t)\} \\
&\quad \cup \{\mathcal{P}^n_{r_4,-r_4^2}(i,j) \oplus n(\sum_{t=1}^{6} m_t)
\end{aligned}$$

$$
\begin{aligned}
& +(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor+n(\sum_{t=1}^{3} r_t)\} \\
f(e_s) \;=\; & \{n(\sum_{t=1}^{6} m_t)+(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor) \\
& +n(\sum_{t=1}^{4} r_t)+i, 1\le i\le (s_1+s_2)+1\}
\end{aligned}
$$

Now we can define the total edge-weights of $G = \text{Amal}(H, P_{s+2}, n)$ constitute the following sets:

$$
\begin{aligned}
w_{f(v_{i,j})} \;=\; & \{\frac{m_1^2n-m_1n}{2}+m_1j\}+\{\frac{m_2^2n}{2}+ \\
& \frac{m_2n}{2}+m_2-m_2j+nm_1m_2\} \\
& +\{\frac{m_3-m_3^2}{2}+m_3^2j+nm_3(\sum_{t=1}^{2} m_t)\} \\
& +\{\frac{m_4}{2}(2m_4n+m_4+1)-m_4^2j+ \\
& nm_4(\sum_{t=1}^{3} m_t\}+\{(\frac{3m_5^2-m_5}{2})j \\
& -m_5^2+m_5+nm_5(\sum_{t=1}^{4} m_t)\}+ \\
& \{\frac{m_6^2(n+2)-m_6}{2}+\lfloor\frac{n+1}{2}\rfloor \\
& (\frac{3m_6^2-m_6}{2})+(\frac{-3m_5^2+m_5}{2})j+ \\
& m_6(n(\sum_{t=1}^{5} m_t)+(m_5-1)\lfloor\frac{n}{2}\rfloor)\} \\
w_{f(v_s)} \;=\; & \{\frac{s_1(m_5^2-1)+s_1^2(3m_5-1)}{2m_5-2}+ \\
& ns_1(\sum_{t=1}^{4} m_t)\}+\{\frac{s_2(m_6^2-1)}{2} \\
& +\frac{s_2^2(3m_6-1)}{2m_6-2}+ns_2(\sum_{t=1}^{4} m_t)\} \\
w_{f(e_{l,j})} \;=\; & \{\frac{r_1^2n-r_1n}{2}+r_1j+r_1(n\sum_{t=1}^{6} m_t \\
& +(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor)\}+\{\frac{r_2^2n}{2} \\
& +\frac{r_2n}{2}+r_2-r_2j+r_2(n\sum_{t=1}^{6} m_t) \\
& +(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor+nr_1)\} \\
& +\{\frac{r_3-r_3^2}{2}+r_3^2j+r_3(n\sum_{t=1}^{6} m_t \\
& +(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor+n\sum_{t=1}^{2} r_t)\} \\
& +\{\frac{r_4}{2}(2r_4n+r_4+1)-r_4^2j+ \\
& r_4(n\sum_{t=1}^{6} m_t+(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor \\
& +n\sum_{t=1}^{3} r_t)\} \\
w_{f(e_s)} \;=\; & \{(s_1+s_2+1)(n\sum_{t=1}^{6} m_t+(m_5+ \\
& m_6-1)\lfloor\frac{n}{2}\rfloor+n\sum_{t=1}^{4} r_t)+ \\
& (s_1+s_2+1)(\frac{s_1+s_2+2}{2})\} \\
W_f \;=\; & w_{f(v_{i,j})}+w_{f(v_s)}+f(A,B)+w_{f(e_{l,j})} \\
& +w_{f(e_s)} \\
=\; & C+j[(m_1-m_2+m_3^2-m_4^2+(\frac{3m_5^2-m_5}{2}) \\
& -(\frac{3m_6^2-m_6}{2})+r_1-r_2+r_3^2-r_4^2]
\end{aligned}
$$

with

$$
\begin{aligned}
C \;=\; & \frac{m_1^2n-m_1n}{2}+\frac{m_2^2n}{2}+\frac{m_2n}{2}+m_2 \\
& +nm_1m_2+\frac{m_3-m_3^2}{2}+nm_3\sum_{t=1}^{2} m_t \\
& +\frac{m_4}{2}(2m_4n+m_4+1)+nm_4\sum_{t=1}^{3} m_t \\
& -m_5^2+m_5+nm_5\sum_{t=1}^{4} m_t+ \\
& \frac{m_6^2(n+2)-m_6}{2}+\lfloor\frac{n+1}{2}\rfloor \\
& (\frac{3m_6^2-m_6}{2})m_6+(n\sum_{t=1}^{5} m_t+(m_5-1) \\
& \lfloor\frac{n}{2}\rfloor)+n(\sum_{t=1}^{6} m_t)+(m_5+m_6-1) \\
& \lfloor\frac{n}{2}\rfloor-1+\sum_{t=1}^{6} m_t+(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor \\
& +\frac{s_1(m_5^2-1)+s_1^2(3m_5-1)}{2m_5-2}+ \\
& ns_1\sum_{t=1}^{4} m_t+\frac{s_2(m_6^2-1)}{2} \\
& +\frac{s_2^2(3m_6-1)}{2m_6-2}+ns_2\sum_{t=1}^{4} m_t+\frac{r_1^2n-r_1n}{2} \\
& +r_1(n\sum_{t=1}^{6} m_t+(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor)+ \\
& \frac{r_2^2n}{2}+\frac{r_2n}{2}+r_2+r_2(n\sum_{t=1}^{6} m_t+(m_5 \\
& +m_6-1)\lfloor\frac{n}{2}\rfloor+nr_1)+ \\
& \frac{r_3-r_3^2}{2}+r_3(n\sum_{t=1}^{6} m_t+ \\
& (m_5+m_6-1)\lfloor\frac{n}{2}\rfloor+n\sum_{t=1}^{2} r_t) \\
& +\frac{r_4}{2}(2r_4n+r_4+1)+r_4(n\sum_{t=1}^{6} m_t \\
& +(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor+n\sum_{t=1}^{3} r_t)+ \\
& (s_1+s_2+1)(n\sum_{t=1}^{6} m_t \\
& +(m_5+m_6-1)\lfloor\frac{n}{2}\rfloor+n\sum_{t=1}^{4} r_t)+ \\
& (s_1+s_2+1)(\frac{s_1+s_2+2}{2})
\end{aligned}
$$

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The total edge-weights of $G = Amal(H, P_{s+2}, n)$ under the labeling f, for $j = 1, 2, \dots, n-1$, constitute the following sets:

$$\begin{aligned} W_f &= \{a, C + 2[m_1 - m_2 \\ &\quad + m_3^2 - m_4^2 + (\frac{3m_5^2 - m_5}{2}) \\ &\quad - (\frac{3m_6^2 - m_6}{2}) + r_1 - r_2 + r_3^2 - r_4^2] \\ &\quad , \dots, C + n[m_1 - m_2 + m_3^2 - m_4^2 \\ &\quad + (\frac{3m_5^2 - m_5}{2}) - (\frac{3m_6^2 - m_6}{2}) \\ &\quad + r_1 - r_2 + r_3^2 - r_4^2]\} \end{aligned}$$

The set of total edge-weights W_f consists of an arithmetic sequence of the smallest value a and the difference $d = \big(m_1 - m_2 + m_3^2 - m_4^2 + (\frac{3m_5^2 - m_5}{2}) - (\frac{3m_6^2 - m_6}{2}) + r_1 - r_2 + r_3^2 - r_4^2\big)$. Since the biggest d is attained when $d = (\frac{3m^2 - m}{2}) + r^2$ then, for $m = p_H - (s+2)$ and $r = q_H - (s+1)$, it gives $0 \leq d \leq p_H^2 + q_H^2 - (s+2)p_H - (s+1)q_H$. It concludes the proof. □

CONCLUSIONS

We have shown the existence of super antimagicness of amalgamation of any graph H, denoted by $G = \text{Amal}(H, P_{s+2}, n)$ for connected graph, by using a partition technique we can prove that $\text{Amal}(H, P_{s+2}, n)$ admits a super(a, d)-H antimagic total labeling for almost feasible difference d. We also note that if the amalgamation of any graph H contains a subgraph as a terminal then finding the labels for feasible d remains widely open. Thus, we propose the following open problems.

Open Problem 1. *Let K be a subgraph of H, does $G = Amal(H, K \subset C, n)$ and $G = tAmal(H, K \subset C, n)$ admit a super $(a, d) - H$ antimagic total labeling for feasible d?*

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