A Zero Crossing-Virus Evolutionary Genetic Algorithm (VEGA) to Solve Nonlinear Equations

M. Ziaul Arif, Zainul Anwar, Ahmad Kamsyakawuni
Mathematics Department, Mathematic and Natural Science Faculty, University of Jember, Jember, Indonesia
e-mail: ziaul.fmipa@unej.ac.id

Abstract—Nonlinear equation is a mathematical problem that is quite difficult to solve. Its analytic solution is not easily discovered. There are several methods used to solve nonlinear equations and the obtained results is in the form of approximation to the analytical solution. Most of the numerical method need appropriate initial value to perform the accuration of the method. However, it will diverge if the initial value is inappropriate. Therefore, we propose discovering the solutions of nonlinear equations by applying metaheuristic methods. In this paper, we present the virus Evolutionary Genetic Algorithm (VEGA) combined with Zero Crossing Method at an early stage to solve nonlinear equations. This study was conducted to test the performance and accuracy of the combined both of the method by providing some examples.

Keyword : Nonlinear equation, VEGA, Zero crossing

INTRODUCTION

A nonlinear equation is one of the problem in mathematics. Searching solution of nonlinear equation is determining the value of x that fulfills the equation of $f(x) = 0$, which is the value of $x = s$, so $f(x) = 0$.

Analytic solution of the nonlinear equation is the best solution of the problem. However, the analytic solution of the nonlinear equation is not easy to be found, but in some cases. So, numeric method becomes the main choice to finish it. Some methods that are generally used to look for the solution of the equation, namely Bisection Method, Newton–Raphson, Regula Falsi, and Secant. Newton–Raphson Method is the most used method to solve the equation because, the count is faster than the others. In the other hand, Newton–Raphson Method can't be used when the first approach point is on the extreme point or top point because, in this point, the value of f'(x)=0 so, the value of denominator $\frac{f(x)}{f'(x)}$ equals to zero.

Beside of numeric method [1-9], nowadays, the solution of nonlinear equation by using metaheuristic method is developed as well. Function optimization becomes the basic development of that method. Some examples of metaheuristic method are Cat Swarm Optimization (CSO) and Genetic Algorithm [10]. Furthermore, there is also Virus Evolutionary Genetic Algorithm (VEGA) which is one of the examples of metaheuristic optimization [10-11]. VEGA is produced by incorporation of genetic algorithm and virus infection. The advantage of using VEGA is being able to get a global optima result]. Whereas, genetic algorithm is stuck on local optima search. Yusuf and Soesanto state that if one population is too small so that, a certain chromosome with some gens that set in the solution will be spread to the other chromosomes. In other words, the first population that is resurrected on the beginning interval may not set in the solution.

One of the methods that be able to cover the disadvantages is zero crossing method. Zero crossing method ensures that in that interval there is a solution by using the change of sign that is located in the end of the closed interval [12].

A problem that will be solved in this research is the application of Zero crossing method and Virus Evolutionary Genetic Algorithm (VEGA) on the solution of nonlinear equation and compare it with some methods that had been observed in the same problem. The purpose of the thing is to know how accurate Zero Crossing-VEGA in solving the equation. The application of Zero Crossing-VEGA is expected to have a better accuracy level in solving the equation. So that, it can give an insight about the application of metaheuristic method in numeric problem from optimization problem perspective.

BACKGROUND

a. Nonlinear Equations

Nonlinear equation is all equations which are not linear equation with changer that has the smallest degree that is equal to one or transcendent and if it is imagined, it may not be straight line. Roots of nonlinear equation can be got analytically and numerically.

In some simple cases, analytic method becomes the main choice, for example, in quadratic polynomial, this formula is used $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. But, the formula cannot be used to look for high degree polynomial solution or transcendent function, so that, numeric method is chosen to search the solution of nonlinear equation.

Numeric solution is done by guessing in sequence, so that, every result is getting more accurate. By doing some procedures, enough iteration, finally, the researcher got estimating result that approach the exact result (the real result) with a false tolerance allowed [1].

b. Zero Crossing

Zero crossing is a condition of one function that has zero value or has a movement from positive to negative value. This method is often used for a requirement of closed method because zero crossing method will evaluate the positive or negative sign from the value of $f(x)$ in the end of interval $(sign(f(a)) \neq sign(f(b)))$. If the value of $f(x)$ in the end of interval have a different sign, so that, the interval indicates that there is the value of $f(x)$ continuous on the interval [12].

c. Virus Evolutionary Genetic Algorithm (VEGA)

Virus Evolutionary Genetic Algorithm (VEGA) is an incorporation between genetic algorithm and virus infection [4]. VEGA is arranged from two populations namely host population and virus population. Host population is equal to the population in the genetic algorithm namely solution candidate. Whereas, virus population is a substring from host population that will infect host population.

According to Fukuda, some elements located in VEGA:

1. Inter infection time: one iteration interval time from virus infection;
2. hostj: host individual for - j before experiencing reverse transcription;
3. hostj': host individual for - j after experiencing reverse transcription;
4. fithostj: the value of host individual fitness before experiencing reverse transcription;
5. fithostj': the value of host individual fitness after experiencing reverse transcription;
6. fitvirusi,j: difference between fithostj and fithostj'

$$fitvirus_{i,j} = fithost_{j'} - fitho_{\ j} \quad (1)$$

7. fitvirusi: virus infection strength
$$fitvirus_i = \sum_{j \in S} fitvirus_{j,i} \quad (2)$$
8. S: set of host individuals that are infected by virus for I;
9. Lifei: Virus life strength
$$Life_i = r \times Lif_{i,t-1} + fitvirus_i \quad (3)$$
10. r: virus life power reduction level. (the value [0,1]);
11. t: virus generation

d. Zero Crossing-VEGA

The procedure of this method:

1. Parameter initialization
   Determining the values of parameter that were needed in the form of host pop size, virus pop size, inter infection time, Pc, Pm, and Pv, function and interval as well.
2. First population generation
   Generating the first population randomly on the interval which was the result of zero crossing as much as host pop size which was a solution candidate. Generating the virus population as many as virus pop size as well.
3. Binary coding
   Converting each host population and virus in the binary form with requirement that the length of virus population bit was shorter than the length of host population bit.
4. Fitness value evaluation $f(x)$
   Evaluating the value of host population fitness by substituting every host to the absolute function value or nonlinear equation. The best solution was determined from the most minimum fitness value.
5. Selection
   Doing tournament selection by grouping some hosts into one tournament. Each tournament would produce a winner from host which had the smallest fitness value. The result of selection was in the form of prospective parent crossover host.
6. Crossover
   The crossover process was done to get the varies host. Whereas, the determination of gens position crossover was done randomly so that, the output offspring could have a good quality, worse, or similar to the parent. The result of prospective parents' selection was chosen randomly based on random numbers that were resurrected. If the random numbers were located in the bottom of PC so that, the prospective parent would be chosen being process parents crossover host. Crossover method used was flat crossover. The result of crossover was determined based on equation of (2.4).
   $$x^1_i = r_i x_i + (1 - r_i) x_{2,i} \quad i = 1..n \quad (4)$$ [3]
7. Mutation
   This process was done to replace some lost hosts during selection process so that it could be examined on the new condition. The count of mutation parent was determined from the result of multiplication between Pm and host pop size and than taken randomly.
8. Interinfection time
   Updating the host population and the fitness value. The number of population had to be equal to host pop size, so that, there were random choices as many as host pop size. Than, checking the condition of interinfection time, if it had been fulfilled so that, continue the virus infection process. If it was not fulfilled yet, repeat the steps (e to g)
9. Virus infection
   Virus infection process was done by changing the infected substring host with virus bit. The infected host number determination was determined from the result of multiplication between Pv and host pop size. Than, choosing randomly as many as the infected host number. Furthermore, the virus infection process was done to get a new virus for the next iteration, the process name is transduction.

The steps of zero crossing-VEGA can be illustrated in a flowchart below:

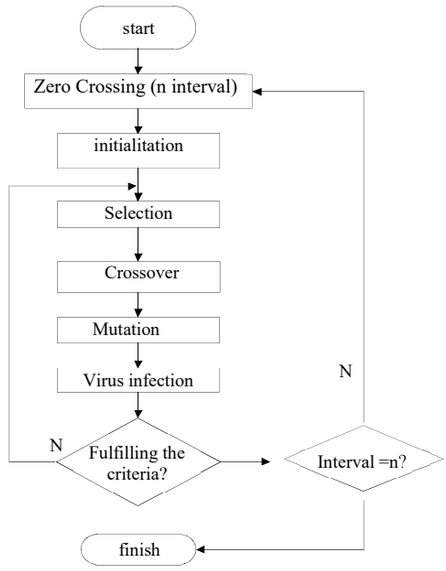


Fig 1. Flowchart Zero Crossing-VEGA

DISCUSION

On this research, there were 11 single root nonlinear equation and 5 double root nonlinear equations that were got from some references referenced. To solve nonlinear equation by using zero crossing-VEGA, the researcher used host pop size parameter = 20, *interinfection time* = 3, *virus pop size* = 9, *Pc* = 0,8, *Pm* = 0,1, *Pv* = 0,4 interval = [-10,10] and iteration = 200.

Example 1:

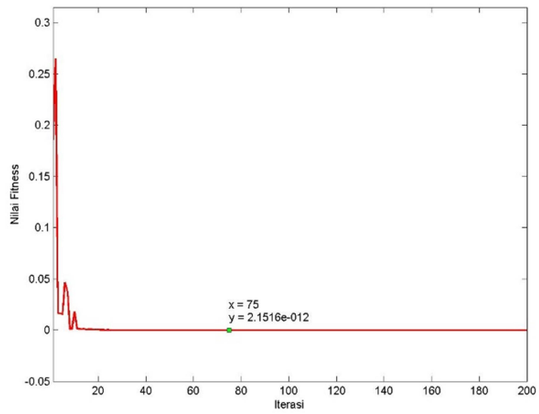
$$\sin^2 x - x^2 + 1 = 0$$

The solution of single root PNL (example 1) is $x = \pm 1{,}40449164821534$. table 1 shows that the result of solution comparison that is got in this research with the solution that is got in the referent journal. Table 1. the comparison of example 1 solution.

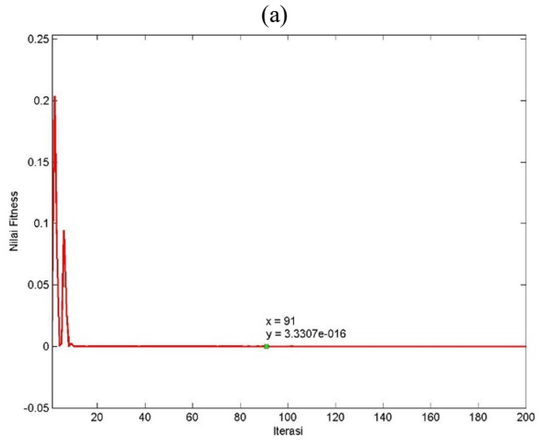
Table 1. Comparison of solution ex.1

Method	Solution
Newton Method	1,4044916482153412260350868178
Zero Crossing-VEGA	1,404491648216208 −1,404491648215341

Zero crossing-VEGA gets 2 solutions with the value of fitness is $|f(x)| = 2{,}1516e-012$ in the first solution and $|f(x)| = 3{,}3307e-016$ for the second solution. The result is better than newton Method that is got 1 solution only. Figure 2 shows that convergent curve fitness value example 1.



(a)



(b)

Fig 2. (a) & (b) Convergent curve of fitness value (ex. 1)

Example 2 :

$$cos\ x - x = 0$$

Solution of single root PNL (example 2) is $x = 0{,}739085133215160641655312087 67$. Table 2 shows that the result of solution comparison that is got in this research with solution that is got in the referent journal.

Table 2. Comparison of solution ex.2

Method	Solution
Noor Method	0,739085133215160641655312087 67
Zero Crossing-VEGA	0,739085133215161

Solution that is got by using zero crossing-VEGA is equal to the solution that is got Noor Method. The fitness value that is got is good $| f(x) | = 0$. Figure 3 shows that convergent curve of fitness value example 2.

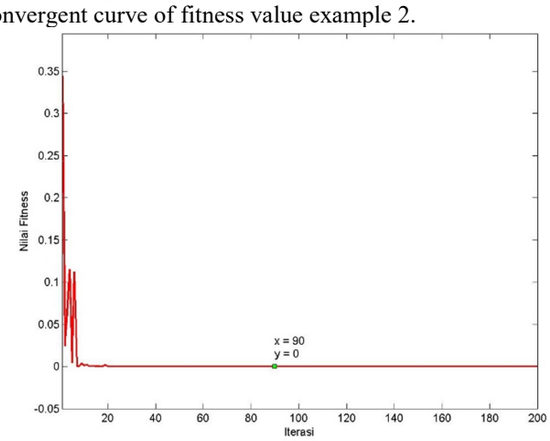


Fig 3. Convergent curve of fitness value (ex. 2)

Example 3 :

$$x^3 - 10 = 0$$

Solution of single root PNL example 3 is $x = 2,154434690031883721759293566 5$. Table 3 shows that the result of solution comparison that is got in this research with solution that is got in the referent journal.

Table 3. Comparison of solution ex.3

Method	Solution
Chun Method	2,154434690031883721759293566 5
Zero Crossing-VEGA	2,154434690031884

Solution that is got by using zero crossing-VEGA is equal to the solution that is got Chun Method. The fitness value that is got is good enough namely $| f(x) | = 1{,}7764e-015$. Figure 4 shows that convergent curve of fitness value example 3.

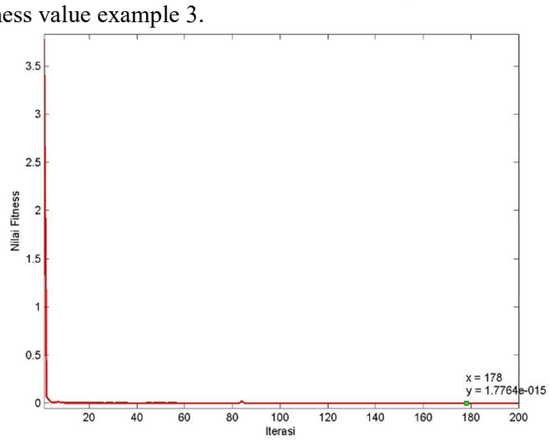


Fig 4. Convergent curve of fitness value (ex. 3)

Example 4 :

$$e^x - 3x^2 = 0$$

Table 4 shows that the result of solution comparison that is got in this research with solution that is got from referent journal.

Table 4. Comparison of solution ex. 4

Method	Solution
Golbabai Javidi Method	−0,45899296202335 0,91001094056187 −0,458962267536945
Zero Crossing-VEGA	0,910007573664188 3,733079028632809

Zero crossing-VEGA gets 3 solutions with the value of fitness is $| f(x) | = 1{,}8677e-015$ for the first solution, $| f(x) | = 1{,}839e-011$ for the second solution and $| f(x) | = 7{,}1054e-015$ for the third solution. The result is better than Golbabai Javidi Method that get 2 solutions only. Figure 5 shows that convergent curve of fitness value from the third solution.

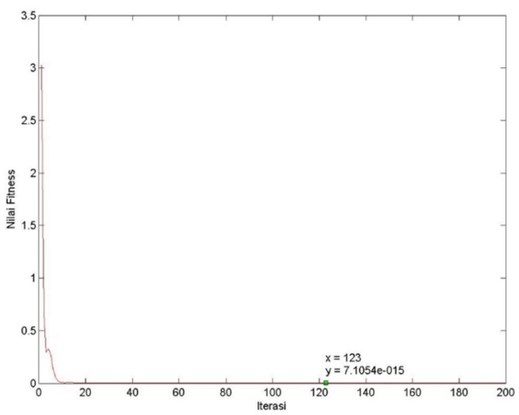


Fig 5. Convergent curve of fitness value (ex. 4)

Example 5 :

$$(ln\ (x^2 + 3x + 5) - 2x + 7)^8 = 0$$

In the double root nonlinear equation, input in the form of function will be transformed by using the equation.

$$F(x) = \frac{f^2(x)}{f(x) - f(x - f(x))}\ [25] \qquad (5)$$

Then, the result of transformation will be processed in the zero crossing level to get interval that is contained solution. Whereas, VEGA level use the input in the form of $f(x)$. Table 5 shows that the result of solution

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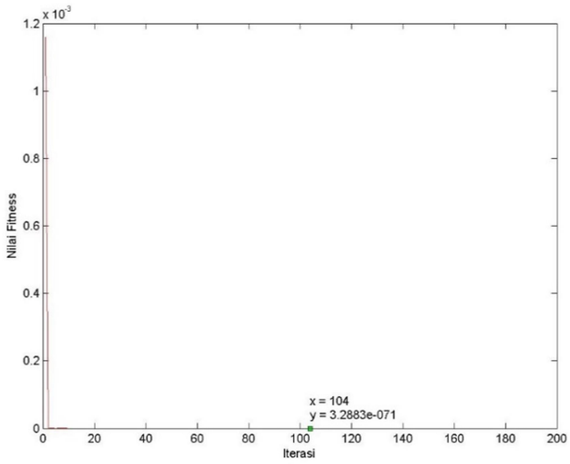
comparison that is got in this research with solution that is got from referent journal.

Table 5. Comparison of solution ex.5

Method	Solution
Newton Method	1,404491648215341226035086817 8
Zero Crossing-VEGA	1,404491648223484 −1,404491648216208

The result that is got is only the value of nonlinear equation root without multiplicity or the count of solutions from the equation, but the value of fitness that is got is better namely $| f(x) | = 3{,}2883e-071$. Figure 6 shows that convergent curve of double root fitness PNL value of example 5.



Figu 6. Convergent curve of fitness value (ex. 5)

$$\left(e^{-x^2+x+3} - x + 2\right)^9 = 0$$

Similar to example 5, the result that is got in the example 6 only in the form of value from equation root without multiplicity. However, the result is equal to the result that is got from the referent journal. Table 6 and figure 7 shows that the result of solution comparison that are got in this research with solution that is got from the referent journal and the convergent curve of double root PNL fitness example 6.

Table 6. Comparison of solution ex.6

Method	Solution
Newton Method	1,404491648215341226035086817 8
Zero Crossing-VEGA	1,404491648223484 −1,404491648216208

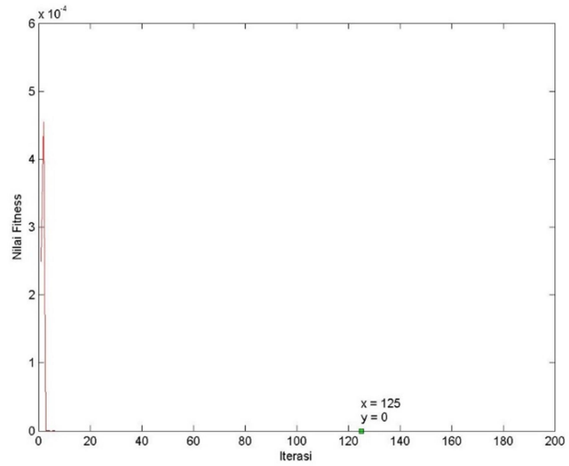


Fig 7. Convergent curve of fitness value (ex. 6)

Example 6 :

No.	PNL	Numerical methods		Zero Crossing VEGA	
		Initial Value	Numerical solution	VEGA Solution	fitness value
1	$\sin^2 x - x^2 + 1 = 0$ [6]	$x_0 = 1$	1,404491648215341226035086817 8	1,404491648223484	$2{,}0214e-011$
				−1,404491648216208	$2{,}1516e-012$
2	$x^2 - e^x - 3x + 2 = 0$ [7]	$x_0 = 2$	0,257530285439860760455367304 94	0,257530285496045	$2{,}12303e-010$
3	$\cos x - x = 0$ [8]	$x_0 = 1{,}7$	0,739085133215160641655372087 67	0,739085133215161	$4{,}409e-016$
4	$(x-1)^3 - 1$ [8]	$x_0 = 3{,}5$	2	2	*Zero Crossing*
5	$x^3 - 10 = 0$ [7]	$x_0 = 1{,}5$	2,154434690031883721759293566 5	2,154434690031884	$1{,}7764e-015$
6	$xe^{x^2} - \sin^2 x + 3\cos x + 5 = 0$ [6]	$x_0 = -2$	−1,207647827130918927009416758 4	−1,207647827131041	$2{,}4722e-012$
7	$e^{x^2+7x-30} - 1 = 0$ [7]	$x_0 = 3{,}5$	3	3 dan −10	*Zero Crossing*
8	$x - 2 - e^{-x} = 0$ [9]	$x_0 = 2$	2,120028238987 64	2,120028238990926	$3{,}6792e-012$
9	$x^2 - (1-x)^5 = 0$ [9]	$x_0 = 0{,}2$	0,345954815848 24	0,345954815848245	$4{,}5242e-015$
10	$e^x - 3x^2 = 0$ [9]	$x_0 = 0$	−0,458992962023 35	−0,458962267536945	$1{,}8677e-015$
		$x_0 = 0{,}5$	0,910010940561 87	0,910007573664188	$1{,}839e-011$
				3,733079028632809	$7{,}1054e-015$
11	$x^3 + 4x^2 - 10 = 0$ [7]	$x_0 = -0{,}3$	1,365230013414096845760806829 0	1,365230013414090	$1{,}066e-013$

(a)

No.	PNL	Numerical methods		Zero Crossing VEGA	
		Initial Value	Numerical solution	VEGA Solution	fitness value
1	$(x^3 + 4x^2 - 10)^3 = 0$ [10]	$x_0 = -0{,}3$	1,365230013414096845760806829 0	1,365230013414097	$4{,}4842e-044$
2	$\frac{(x-\sqrt{5})^4}{(x-1)^2+1} = 0$ [11]	$x_0 = 1{,}9$	2,236067977499790	2,236067980527878	$3{,}326e-035$
3	$(\ln(x^2 + 3x + 5) - 2x + 7)^8 = 0$ [11]	$x_0 = 34; 5; 5{,}8$	5,4690123359101421	5,469012336805463	$3{,}2883e-071$
4	$(e^{(-x^2+x+3)} - x + 2)^9 = 0$ [11]	$x_0 = 2{,}5; 18;$	2,4905398276083051	2,490539827608305	0
5	$(e^{-x} + 2\sin x)^4 = 0$ [11]	$x_0 = 4; 2{,}5$	3,1627488709263654	3,162748870926407	$5{,}2803e-053$

(b)

Fig 8. The comparison of the nonlinier equation result (a) single root (b) even roots from zero crossing-VEGA and numeric method that had been observed.

Based on table 7 (a) and (b), zero crossing-VEGA can be able to give the value of fitness that is near to zero and there is also a value that is similar to exact solution. This thing indicates that roots of nonlinear equation both single root and double root have been found nicely. Furthermore, proposed methods in this research is more effective than Newton-Raphson method that need the first correct value because, the first value error can cause no convergent result. Another advantage is nonlinear equation derivative is not needed so that, it is able to search from liner equation that is the derivative is hard to be found. Besides, the proposed method has a disadvantage like the time process is longer than Newton-Raphson because it depends on some random numbers in the process.

The researcher not only applies zero crossing-VEGA, but also observes the influence of some parameters on the got solution and running time. Some parameters value gives an effect on the count time of using the program, however, there is also influence that gives effect on the solution whether there is a value change. The length of the count process(time) has each parameter so that, the count process will need a long time. Whereas in the Pc parameter value, the kind of thing is not available, the count time cannot be determined from the big or small parameter value. But , Pc parameter value gives the best solution for this researche problem in the value of 0,8. Inter infection time parameter value, pop size virus, Pm, and Pv in sequence give the best solution with the value of 3 , 9 , 0,1 , 0,4.

CONCLUSION

Based on the explanation above, it can be concluded that the zero crossing-VEGA can solve nonlinear equation with a good accuracy altough it needs a time to find nonlinear equation solution. The parameter value that is very optimal for this researche problem are interinfection time = 3, *pop size* = 20, *pop size virus* = 9, *Pc* = 0,8 , *Pm* = 0,1 and *Pv* =0,4.

REFERENCES

[1] B. Triatmodjo, “Metode Numerik”, Yogyakarta: Peta Offset, 1996.

[2] T. Fukuda, K. Shimojima and N. Kubota, “Virus Evolutionary Genetic Algorithm and Its Applications to Traveling Salesman Problem”, Editor: Xin Yao, Singapura: World Scientific, 1999.

[3] J. M. Mendes, “A Comparative Study of Crossover Operators for Genetic Algorithms to Solve the Job Shop Scheduling Problem”, Portugal: School of Engineering – Polytechnic of Porto, 2013.

[4] P. Wang, “A Third-Order Family of Newton Like-Iteration Methods for Solving Nonlinear Equations”, Journal of Mathematics and Stochastics, vol. 3, issue 1, pp. 13-19, 2011.

[5] C. Chun, “Iterative Methods Improving Newton’s Method by Decomposition Method”, Computer and Mathematics with Aplication, vol. 50, pp. 1559-1568, 2005.

[6] M. A. Noor, “New Iterative Schemes for Nonlinear Equations”, Aplied Mathematics and Computation, vol. 187, pp. 937-943, 2007.

[7] M. Javidi and A. Golbabai, “A Third-Order Newton Type Method for Nonlinear Equations Based on Modified Hemotopy Peturbation Method”, Aplied Mathematics and Computation, vol. 191, pp. 199-205, 2007.

[8] L. Shengguo, L. Xiangke and C. Lizhi, “A New-Fourth Order Iterative Method for Finding Multiple Roots of Nonlinear Equations”, Aplied Mathematics and Computation, vol. 215, pp. 1288-1292, 2009.

[9] J. Liang, et al., “Fifth-Order Iterative Method for Solving Multiple Roots of the Highest Multiplicity of Nonlinear Equation”, Algorithms, vol. 8, pp. 656-668, 2015.

[10] A. Yusuf and O. Soesanto, “Algoritma Genetika Pada Penyelesaian Akar Persamaan Sebuah Fungsi”, Jurnal Matematika Murni dan Terapan, vol. 6, issue 2, pp. 047–056, 2012.

[11] N. A. Fountas and N. M. Vaxevandis, “A Modified Virus Evolutionary Genetic Algorithm for Rough Machining Optimization of Sculptured Surfaces”, International Journal of Engineering Tome XI, vol. 3, pp. 283 – 288, 2013.

[12] F. Zhang, M. Yeddanapudi and P. J. Mosterman, “Zero Crossing Location and Detection Algorithm for Hybrid Simulation”, USA: The Mathworks, Inc, 2008.