# Direct Scattering Problem for Microwave Tomography

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**Abstract**— The Direct scattering problem is composed in a pair of integral equations. The equivalence current density and the ratio of dielectric contrast are set as variables by applying volume equivalence principle. The problem is solved using the method of moment (MM). The MM solutions are compared to the exact solutions. The results show that the MM solution is accurate. Perfect solution is generated for scattering from a wide range dielectric contrast. But, it is sensitive to cell's size. A good result is provided at comparably small cell's size..

**Keywords**— Direct scattering problem; The Method of moment; Microwave tomography.

## INTRODUCTION

Direct scattering problems are essential for developing methods in inverse scattering problems as in microwave imaging. The accuracy of the direct problems directly influences the results of the reconstruction image.

A general scattering problem solved using integral equations method gains popularity for the last few decades. The magnetic and electric field interior and exterior of a cylinder with arbitrary cross-sectional shape initially were simulated by Richmond [1]. Harington [2] proposed a numerical solution of integral equation using the method of moment (MM). The MM quantified current distribution in the surface of conducting cylinder. The integral solutions can be implemented in surface integral and volume integral.

The application of surface integral usually involve scattering from conducting cylinder. Various researches have been conducted. The MM has been implemented to simulate scattering from various types of PEC problems [3][4]. The surface integral can be applied to calculate the scattering from conducting object of homogeneous scatterer but it is not suitable for inhomogeneous penetrable object. Thereto the volume integral is put as the topic in this paper.

A wide area of applications of volume integral equation was investigated, which included antenna [5], transmission line [6], flow metering [7], imaging [8][9] and scattering [10]. The development of the integral equations problems includes the formulation of the volume integral [11] and hybrid of surface-volume integrals [12]. In the same time the solution of the equations is grown. The method of moment (MM) has been developed rapidly since the work of Richmond's studies in 1965 [1]. It becomes very popular as a solver for integral equations. The MM is a numerical procedure to solve a linear operator equation by transforming it to a system of simultaneous linear algebraic equation which is commonly referred to as matrix equations.

The volume integral works in the centre of the cells across the object of interest (*OI*) sliced. This is suitable for microwave imaging that reconstructs the image of *OI* cross section. Some works had been done in this topic. Richmond approach was used to develop the forward problems [13][14][15][16]. Then it was followed by developing several inverse methods base on the defined forward problems [8][9][10]. The results show that the volume integral can be used in developing microwave imaging. It can be used to reconstruct simple object. Nevertheless, it is sensitive to a noise and initial guess beside gaining big error, thereto, the investigation on the forward problems needs to be done.

The feature of MM has included a frequency domain prediction technique and taken to the account the entire electromagnetic phenomenon and the polarization effects for excite field. The MM, which is based on integral equation technique, advances in the accuracy of the results as it is essentially exact and provides direct numerical solutions. It is also applicable to complex inhomogeneous *OI*. Nevertheless, the MM is classified into low frequency methods. It is typically limited to problems of small electrical size due to limitations of computation time and system memory. Thereto, an investigation of MM solution for higher frequency is necessary to be done.

## THEORY

The direct scattering from inhomogeneous problem involves the interaction between microwave and penetrable object. The interaction can be described in term of volume electric field integral equations (VEFIE) by applying volumetric equivalence principle. The integral enforces the electric field inside the domain object as

$$\mathbf{E}^{\mathrm{i}}(\vec{r}) = \mathbf{E}^{\mathrm{t}}(\vec{r}) - \mathbf{E}^{\mathrm{s}}(\vec{r}), \vec{r} \in V_d \quad (1)$$

In the case of microwave signal illuminate a material that is composed of dielectric material, the electric vector **F** in VEFIE vanishes as magnetic vector **A** can be canceled out in MFIE case. Thus, the VEFIE can be written as

$$\mathbf{E}^{\mathrm{i}}(\vec{r}) = \mathbf{E}(\vec{r}) + \frac{j\eta}{k}\left[k^2\mathbf{A}(\vec{r}) + \nabla\left(\nabla.\mathbf{A}(\vec{r})\right)\right] \quad (2)$$

Where $\mathbf{A} = \int_{\mathrm{V}} \mathbf{J}_v(\vec{r})G(r,r')ds'$

The equivalent current density $\mathbf{J}$ is divined as $\mathbf{J} = j\omega\varepsilon_0(\varepsilon_r - 1)\mathbf{E}$ by applying volumetric equivalent principle. The dielectric constant $\varepsilon_r$ which is formed in complex number describes an inhomogeneous object. In two dimensional domains, the dielectric is taken to be constant $[\varepsilon_r(x,y)]$ in each cell.

Assuming that a TM wave illuminates the 2D object. The incident field propagates in z direction $\left(E_z^i\right)$ and the $J$ is not varied that is $(\nabla.J) = 0$. The integral equations turn into simpler form as the second order deferential can be eliminated. Furthermore a dielectric contrast ratio is introduced as

$$\chi_\varepsilon = \frac{\varepsilon_r - 1}{\varepsilon_r} \quad (3)$$

The dielectric contrast of the background should be eliminated as if the volume equivalence is applied. The electric field is zero at the background and the equivalent current density will be infinity. For direct problems the background cannot be separated from the object as the object dielectric, size and position are the unknown variables for microwave imaging problems. Thus, the contrast ratio is employed to avoid zero division. Then, the integral equations can be written as

$$E_z^s = -\frac{k\eta}{4}\iint J_z(x',y')H_0^{(2)}(kr)dx'dy' \quad (4)$$

$$\chi_\varepsilon(x,y)E_z^i(x,y) = \mathrm{c}_1 J_z(x,y) - \mathrm{c}_2\sum_{\mathrm{n=1}}^{\mathrm{N}}\left[\chi_{\varepsilon nm}\iint J_z(x',y')H_0^{(2)}(kr)dx'dy'\right] \quad (5)$$

The constants $\mathrm{c}_1$ and $\mathrm{c}_2$ can be determined analytically by multiplying the ratio into integral equation. The domain is divided into N number of cells equal in volume. Then, the moment of method is applied using basis to construct a matrix equations. If the cell size is approximated by a circle of the same area with radius a, the integral of Hankel's function can be evaluated analytically as

$$\int_{\emptyset'=0}^{2\pi}\int_{r'=0}^{a} H_0^2(k\rho)r'dr'd\emptyset' = \begin{cases} \frac{2\pi}{k}J_0(k\rho)H_1^2(ka) - \frac{4j}{k^2} & (\rho < a) \\ \frac{2\pi}{k}J_1(ka)H_0^2(k\rho) & (\rho > a) \end{cases} \quad (6)$$

Then, the matrix form of the integral equations can stated in a pair of matrix equations as follows:

$$\mathbf{E}_s = -\mathbf{Z}_{mn}\mathbf{J} \quad (7)$$

$$\boldsymbol{\chi}\mathbf{E}_i = [\boldsymbol{\chi}\mathbf{Z}_{nn} - \mathbf{C}\,]\mathbf{J} \quad (8)$$

The $\mathbf{Z}_{mn}$ is the linear operator of the object's cells with antennas, $\mathbf{Z}_{nn}$ is the operator of the object's cells with them self, and **CI** is a diagonal matrix constant of *n x n* in size. The direct problem is done by solving the matrix equation (7) and (8) simultaneously.

## METHODS

Comparative study is used to analyze the accuracy of the MM solutions. The MM solutions are compared to the exact solutions. A simple cylindrical geometry object with real dielectric contrast is used as it is the only possible geometry that exact solutions can handle. The object of interest ($OI$) is defined as infinite cylindrical object with $a_{OI}$ −radius and real $\varepsilon_r$ dielectric. The exact solutions calculate the scattering fields in observation domain ($O$) using cylindrical harmonic expansions. The MM approaches the cylindrical in squared meshes.

In 2D view the *OI* is placed in square area which is divided in $N_A$ small squared area with $a_{cell}$ −radius equivalent. The radius of the cell is varied by changing the $N_A$. The dielectric of the cells which are placed inside *OI* is set as $\varepsilon_r$ others are set as 1. The number of cells inside *OI* is labeled as $N$. The size of the cells is equal among them. The size must be small compared to the wave length. Inside material the wave length is defined as $\sqrt{|\varepsilon_r|}\lambda_{OI} = \lambda_0$. Peterson [17] recommends the minimum number of cells for homogeneous dielectric cylindrical cross-section is $100cells/\lambda_{OI}^2$. This is approximately similar to $\approx 0.05\lambda_{OI}$-cell radius.

The results of MM solutions and exact solution are compared. The exact solutions is analytically derive using harmonic expansion as in [18]. An absolute mean error is used to measure the quality of MM solutions relative to exact solutions. the quality of the solution is examined in term of cell size and dielectric contrast variations.

## NUMERICAL RESULT

Fig 1 shows the scattered fields for various cell's size. A 3.0 GHz plane wave illuminates a cylindrical dielectric cylinder with diameter of $2/3\lambda_0$. The dielectric of the cylinder is 4. Three *OI* models are selected and applied to the MM solutions. the models diver is the number of the cells. This means that the size of the cells are varied among them. The scattered fields in $O$ domain are simulated at 64 antennas. It can be seen that $0.08\lambda_{OI}$-cell radius produce relatively big error. Reasonable good result is generated by model with $0.06\lambda_{OI}$-cell radius, and $0.03\lambda_{OI}$-cell radius produce excellent scattered fields. it can be seen that the error of the fields are dependent on the position of the Rx antennas relative to Tx. The longest and shortest distance of Rx and Tx produce greater errors compared to other position of Rx antennas.

Fig 2 shows the accuracy of the MM solutions at two observation points for various models. The models are varied by dividing the background of *OI* which is constant in size of $1/2\lambda_0$ in 10 up to 10 000 cells. The working frequency is kept constant at 1.5 GHz. The bigger the number of cells is the smaller the cell's size compared to $\lambda_0$. It can be seen that models with radius $\leq 0.05\lambda_0$ produce a good approximation to exact solutions. The MM solution approaches the exact solution. It can be seen that the error fluctuates and increases as the cell's size is enlarged. The 5% error is produced in models at $a \approx 0.05\lambda_0$. The size follows the recommendation.

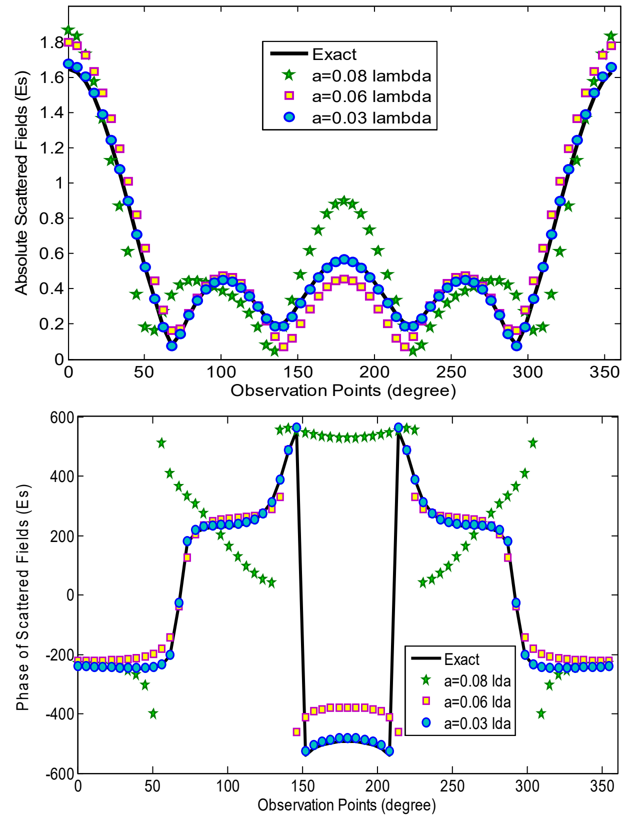


Figure 1 – The magnitude (a) and the phase (b) of the scattered fields of the MM solutions and exact solution for scattering by dielectric cylinder with $\varepsilon_r = 4$ at three different cell size. The antennas are placed at 5 $\lambda_0$. A plane wave is transmitting from ($\rho = 5\lambda_0, \phi = 0$) and the scattered fields are receiving at 64 antennas ($\rho = 5\lambda_0, \phi = 0..360$).

Decreasing the cell's size will enlarge the matrix size. The computational time improves significantly as the cell number reaches 60x60 object divisions. Iteration for calculating the Green's functions between cells and the size of full matrix which consist of complex numbers are burdening the computation process. A 2G Bytes RAM computer can handle only 50x50 *OI*. A supercomputer is needed to handle a bigger number of cells. Improving cell size at some point does not improve the quality. Test 4 at table 1 which is developed from model at test 3 by increasing the cell's number produce relatively similar error. This proceeds to an idea that wise selection of cell's number should be put into account for the design of models.

Table 1 – Scattering obtained from homogeneous dielectric cylinder using MM at various cell's size

| No | N | a | Abs (E) MM Solution | Abs (E) Exact | Phase (E) MM Solution | Phase (E) Exact |
|---|---|---|---|---|---|---|
| 1 | 177 | 0.126 | 0.191 (16.83%) | | 2.793 (55.13%) | |
| 2 | 316 | 0.093 | 0.211 (07.92%) | 0.229 | 2.219 (23.23%) | 1.801 |
| 3 | 716 | 0.063 | 0.231 (00.57%) | | 1.864 (03.55%) | |
| 4 | 1264 | 0.046 | 0.225 (01.83%) | | 1.868 (03.75%) | |

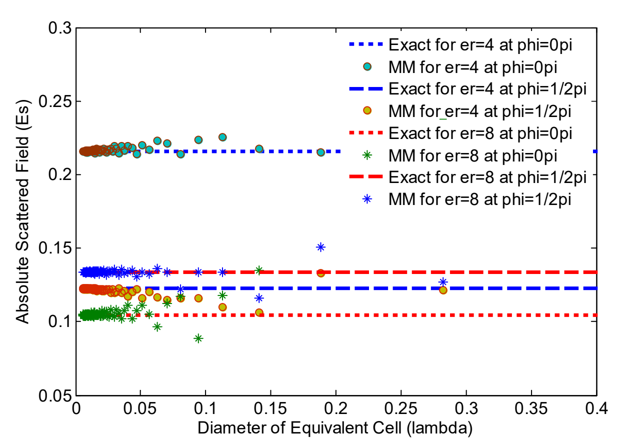


Figure 2 – Comparison of Scattered fields resulted by exact solutions and MM solutions at two different points of ($\rho =$

$+5\lambda_0, \phi = 0)$ and $(\rho = +5\lambda_0, \phi = 1/2\pi)$ for various equivalent diameter of cell.

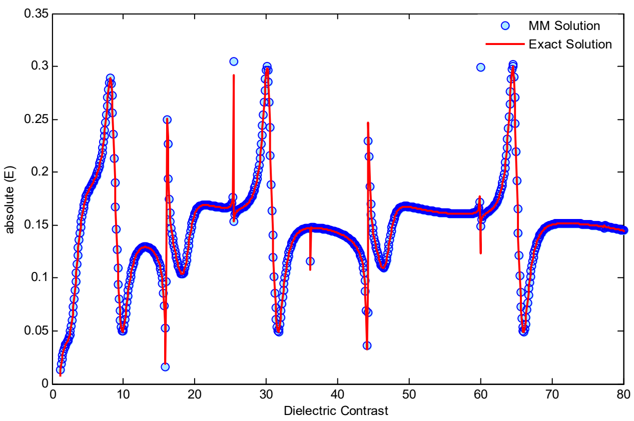


Figure 3 – Comparison of exact solutions and MM solutions of scattering field at a single point of $\rho = +5\lambda_0, \phi = 0$ for various dielectric contrasts

The effect of dielectric contract to the accuracy of MM solutions is shown in fig 3. The data are taken at a single point in $+x$ direction. The OI size is taken to be constant at $a = 0.05\lambda_0$. The object model is divided in $N_A = 20x20$ to guarantee the cell size meets the recommendation size. It can be seen that the absolute of the scattering field resulted from of MM solutions is close to the exact solutions. The MM is able to handle the dielectric variations. It can be used to reconstruct scattering field from high contrast dielectric object. In average the error is very small. In several points the errors significantly increase. The problem appears in place where the shape of the field dramatically changes. The complexity and pattern change is hard to solve. This can be seen in figure 3 that the MM solution misses the red line of exact solutions at the sudden changed points.

## CONCLUSION

The numerical solution for microwave scattering is set as forward problems. The problem is derived to solve Maxwell's equations in frequency domain. The electric field integral equation in transverse magnetic polarization is selected as the problems. Volume integral equations are derived for inhomogeneous cases. The ratio of dielectric contrast and equivalent current density are selected as variable to be solved. This creates an alternative solution for volume integral as the infinite solution at the background can be eliminated without canceling the background cells. This produces a flexibility in developing the inverse problems for microwave imaging.

The solutions are posed in the centre of the cells. The MM is applied to solve the integral equations. Using pulse basis function, the integral equation is transferred into linear equations in term of matrix equations. The matrix equation is derived in two different relations. The unknown equivalent current density in each cell is calculated using known incident fields and the ratio of dielectric contrast. The equivalent principle is applied to replace the unknown variable. The unknown scattered field is calculated using the equivalent current density. The numerical solutions are applied to simple problems. The results are compared to exact solutions. The results show that the pulse basis function demonstrates the ability to expand the current density J. It can be seen that the result depends on the cell size. The size of the cells should be much smaller than the wave length. The MM solutions in J formulations show good accuracy. The MM solves VEFIE in a very high range of dielectric contrast.

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