SUPER (a,d)-EDGE ANTIMAGIC TOTAL LABELING OF CONNECTED TRIBUN GRAPH

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Abstract. *A G graph of order p and size q is called an (a,d)-edge antimagic total if there exist a bijection f:V(G)∪E(G)⟶{1,2,…,p+q} such that the edge-weights, w(uv)=f(u)+f(v)+f(uv), uv∈E(G), form an arithmetic sequence with first term a and common difference d. Such a graph G is called super if the smallest possible labels appear on the vertices. In this paper we study super (a, d)-edge-antimagic total properties of connected Tribun graph. The result shows that a connected Tribun graph admit a super(a,d)-edge antimagic total labeling ford=0,1,2 for n≥1. It can be concluded that the result of this research has covered all the feasible n,d.*

Key Words: *(a,d)-edge antimagic vertex labeling, super(a,d)-edge antimagic total labeling, Tribun Graph.*

INTRODUCTION

In mathematics and computer science, graph theory is the study of graphs, mathematical structures used to model pairwise relations between objects from a certain collection. A ”graph” in this context refers to a collection of vertices or ’nodes’ and a collection of edges that connect pairs of vertices. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another.

Graphs are one of the prime objects of study in discrete mathematics. There are various types of graph labeling, one is a super (*a; d*)-edge antimagic total labeling (*SEAT*). This problem is quite difficult as assigning a label on each vertex, in such a way it has a weight set in which the elemen has the same different, indicate a big problem, and there is no guarantee if a graph G has a super (*a; d*)-edge antimagic total labeling, then the disjoint union of graph *G*has super (*a; d*)-edge antimagic total labeling as well.

In this paper we investigate the existence of super (a,d)-edge-antimagic total labelings for connected graphs. Some constructions of super (a,d)-edge antimagic total labelings for *mLn* and *mLi;j;k* have been shown by Dafik, Slamin, Fuad and Rahmad and super (a; d)-edge-antimagic total labelings for Generalized Petersen have been described by Debby in. Dafik et al also foundsome families of graph which admits super

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(*a; d*)-edge antimagic total labelings, namely *mCn;mPn;mKn; n : : : n* and *m* in [3*;* 4*;* 5]. We will now concentrate on the connected Tribun denoted by$\mathfrak{I}_n$.

Tribun Graph

Tribun graph denoted by $\mathfrak{T}_n$ with $n \geq 1$ is a connected graph with vertex set. Tribun graph have *vertex*, $V = \{B, x_i, z_j, y_i; 1 \leq i \leq n; 1 \leq j \leq 2n+1; n \epsilon N\}$ and *edge*, $E = \{Bz_1, Bz_2, Bz_3 \cup z_j z_{j+1}; 1 \leq j \leq n \cup x_i z_{2i+1}; 1 \leq i \leq n \cup x_i z_{2i+3}; 1 \leq i \leq n \cup y_i z_{2i+1}; 1 \leq i \leq n \cup y_i z_{2i-1}; 1 \leq i \leq n\}$. Thus $|V(\mathfrak{T}_n)| = p = 4n + 2$ and $|E(\mathfrak{T}_n)| = q = 8n + 1$. From the figure 1 gives an example of a $(3, 1)$-edge antimagic vertex labeling of $\mathfrak{T}_n$.

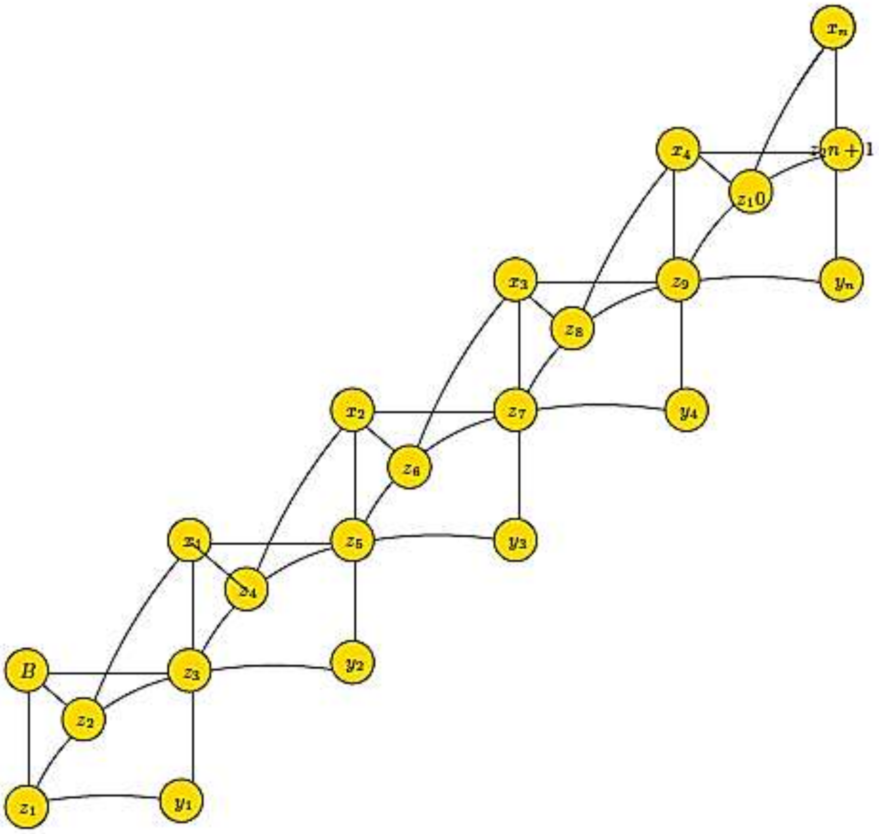

Figure 1: Tribun graph $\mathfrak{T}_n$

Super *(a,d)*-edge Antimagic Total Labeling

An *(a,d)*-edge-antimagic total labeling on a graph G is a bijective function f:V(G) ∪ E(G) → {1,2,…,p+q} with the property that the edge-weights w(uv)=f(u)+f(uv)+f(v), u,v∈ E(G), form an arithmetic progression {a,a+d,a+2d,…,a+(q-1)d}. where a>0 and d ≥ 0 are two fixed integers. If such a labeling exists then G is said to be an *(a,d)*-edge-antimagic total graph}. Such a graph G is called super if the smallest possible labels appear on the vertices. Thus, a super *(a,d)*-edge-antimagic total graph is a graph that admits a super *(a,d)*-edge-antimagic total labeling.

We start this section by a necessary condition for a graph to be super (*a; d*)-edge antimagic total, providing a least upper bound for feasible values of *d*. This lemma can be found in (*Dafik;* 2007: 26 - 27).

Lemma 1 *If a (p,q)-graph is super (a,d)-edge-antimagic total then* $d \leq \frac{2p+q-5}{q-1}$.

Proof. Assume that a *(p,q)*-graph has a super *(a,d)*-edge-antimagic total labeling $f: V(G) \cup E(G) \to \{ 1,2,..., p+q \}$. The minimum possible edge-weight in the labeling *f* is at least 1+2+p+1 = p+4. Thus, a ≥ p+4. On the other hand, the maximum possible edge-weight is at most (p-1)+p+(p+q) =3p+q-1. So we obtain a+(q-1)d ≤ 3p+q-1 which gives the desired upper bound for the difference *d*

Lemma 2 A *(p,q)*-graph G is super edge-magic if and only if there exists a bijective function *f* :V(G)→{ 1,2,…, p } such that the set S={ *f(u)+f(v): uv* ∈ *E(G)*} consists of q consecutive integers. In such a case, *f*extends to a super edge-magic labeling of G with magic constanta=p+q+s, where s = min(S) and S= { a- (p+1), a- (p+2), ….,a-(p+q) }. The two above lemma will be used for develop theorem 1.

RESEARCH METHODS

In this paper, we using a pattern recognition and axiomatic deductive to get the bijection function of super (*a, d*)-edge antimagic total labeling of Tribun graph. The research techniques are as follows: (1) calculate the numberof vertex *p* and size *q* of graph $\mathfrak{T}_n$; (2) determine the upper bound for valuesof *d*; (3) determine the label of *EAVL* (edge-antimagic vertex labeling) of$\mathfrak{T}_n$; (4)if the label of *EAVL* is expandable, then we continue to determine the bijectivefunction of *EAVL;* (5) label the graph Tn with *SEATL* (super-edge antimagic totallabeling) with feasible values of *d* by using Lemma 1 and (6) determine thebijective function of super-edge antimagic total labeling of graph$\mathfrak{T}_n$.

RESULT AND DISCUSSIONS

If Tribun graph, has a super (a,d)-edge-antimagic total labeling then, for p=4n + 2 and q=8n + 1, it follows from Lemma 1 that the upper bound of d is d ≤ 2 or d

∈{0,1,2}. The following lemma describes an (a,1)-edge-antimagic vertex labeling for Tribun Graph.

Lemma 3*If $1 \leq i \leq n$ then the Tribungraph $\mathfrak{T}_n$. has an (8i - 5,1)-edge-antimagicvertex labeling.*

Proof. Define the vertex labeling f_1:$V(\mathfrak{T}_n)f_1$: → {1,2, … ,4n+2}g in the following way:

$$\begin{aligned}
f_1(B) &= 1, \\
f_1(x_i) &= 4i+1, \text{untuk } 1 \ \leq i \leq n \\
f_1(z_j) &= 2j, \text{untuk } 1 \ \leq j \leq 2n+1 \text{ dan j } \epsilon \text{ bilangan ganjil} \\
f_1(z_j) &= 2j-1, \text{untuk } 1 \ \leq j \leq 2n+1 \text{ dan j } \epsilon \text{ bilangan genap} \\
f_1(y_i) &= 4i, \text{untuk } 1 \ \leq i \leq n
\end{aligned}$$

The vertex labeling f_1 is a bijective function. The edge-weights of $\mathfrak{T}_n$., underthe labeling f_1, constitute the following sets

$$\begin{aligned}
w_{f_1}(Bz_1) &= 3, \\
w_{f_1}(Bz_2) &= 4, \\
w_{f_1}(Bz_3) &= 7, \\
w_{f_1}(z_jz_{j+1}) &= 4j+1, \text{untuk}1 \leq j \leq 2n, \\
w_{f_1}(x_iz_{2i+2}) &= 8i+4, \text{untuk}1 \leq i \leq n-1, \\
w_{f_1}(x_iz_{2i}) &= 8i, \text{untuk}1 \leq i \leq n, \\
w_{f_1}(x_iz_{2i+1}) &= 8i+3, \text{untuk}1 \leq i \leq n, \\
w_{f_1}(x_iz_{2i+3}) &= 8i+7, \text{untuk}1 \leq i \leq n-1, \\
w_{f_1}(y_iz_{2i+1}) &= 8i+2, \text{untuk}1 \leq i \leq n, \\
w_{f_1}(y_iz_{2i-1}) &= 8i-2, \text{untuk}1 \leq i \leq n,
\end{aligned}$$

It is not difficult to see that the set $w_{f_1} = \{3, 4, 5, \dots, 8n+3\}$consists ofconsecutive integers. Thus $f_1$is a $(3,1)$ −edge antimagic vertex labeling. Baca, Y. Lin, M. Miller and R. Simanjuntak [13], Theorem 5) have proved that if *(p,q)*-graph G has an *(a,d)*-edge antimagic vertex labeling then G has a super *(a+p+q,d-1)*-edge antimagic total labeling and a super *(a+p+1,d+1)*-edge antimagic total labeling. With the Lemma 3 in hand, and using Theorem 5 from [13], we obtain the following result.

Theorem 1. *If n ≥ 1 then the graph $\mathfrak{T}_n$has a super (12n+6,0)-edge-antimagic total labeling and a super (4n+6,2)-edge-antimagic total labeling.*

Proof.

Case 1. d=0

We have proved that the vertex labelingf_1 is a $(3,1)$ $-$edge antimagic vertexlabeling. With respect to Lemma 2, by completing the edge $labels$ $\{p+1,\ p+2,\dots,p+q\}$, we are able to extend labeling f_1 to a super $(a,0)$ $-$edge-antimagic total labeling, where, for $p=4n+2$ and $q=8n+1$, the value $a=12n+6$.

Case 2. d=2

Label the vertices of $\mathfrak{I}_n$with f_3 that the edge labeling for *d* = 2, so we can that label the edges with the following way.

$$\begin{array}{lcl}
f_3(Bz_1) & = & 4n+3,\\
f_3(Bz_2) & = & 4n+4,\\
f_3(Bz_3) & = & 4n+7,\\
f_3(z_jz_{j+1}) & = & 4n+4j+1, \text{untuk} 1 \le j \le 2n,\\
f_3(y_iz_{2i-1}) & = & 4n+8i-2, \text{untuk} 1 \le i \le n,\\
f_3(x_iz_{2i}) & = & 4n+8i, \text{untuk} 1 \le i \le n,\\
f_3(y_iz_{2i+1}) & = & 4n+8i+2, \text{untuk} 1 \le i \le n,\\
f_3(x_iz_{2i+1}) & = & 4n+8i+3, \text{untuk} 1 \le i \le n,\\
f_3(x_iz_{2i+2}) & = & 4n+8i+4, \text{untuk} 1 \le i \le n-1,\\
f_3(x_iz_{2i+3}) & = & 4n+8i+7, \text{untuk} 1 \le i \le n-1,
\end{array}$$

The total labeling f_1 is a bijective function from$V\,\mathfrak{I}_n \cup E\,\mathfrak{I}_n$onto the set$\{1,2,3,\dots,4n+2\}$. The edge-weights of $\mathfrak{I}_n$, under the labeling f_3, constitute the sets

$$\begin{array}{lcl}
W_{f_3} & = & \{w_{f_3}+f_2(Bz_1)\} = 4n+6\\
W_{f_3} & = & \{w_{f_3}+f_2(Bz_2)\} = 4n+8\\
W_{f_3} & = & \{w_{f3}+f_2(Bz_3)\} = 4n+14\\
W_{f_3} & = & \{w_{f_3}+f_2(z_jz_{j+1});\ \text{jika}\ 1 \le j \le 2n\} = 4n+8j+2\\
W_{f_3} & = & \{w_{f3}+f_3(y_iz_{2i-1});\ \text{jika}\ 1 \le i \le n\} = 4n+16i-4\\
W_{f_3} & = & \{w_{f3}+f_3(x_iz_{2i});\ \text{jika}\ 1 \le i \le n\} = 4n+16i\\
W_{f_3} & = & \{w_{f3}+f_3(y_iz_{2i+1});\ \text{jika}\ 1 \le i \le n\} = 4n+16i+4\\
W_{f_3} & = & \{w_{f3}+f_3(x_iz_{2i+1});\ \text{jika}\ 1 \le i \le n\} = 4n+16i+6\\
W_{f_3} & = & \{w_{f3}+f_3(x_iz_{2i+2});\ \text{jika}\ 1 \le i \le n-1\} = 4n+16i+8\\
W_{f_3} & = & \{w_{f3}+f_3(x_iz_{2i+3});\ \text{jika}\ 1 \le i \le n\} = 4n+16i+14
\end{array}$$

It is not difficult to see that the set $w_{f_3}=\{4n+6,4n+8,\dots,2on+6\}$contains an arithmetic sequence with $a=4n+6$ and $d\ =\ 2$. Thus $f_3$is a$super$ $(4n+6{,}2)$ $-$edge-antimagic total labeling. This concludes the proof.

Theorem 2. *If n ≥ 1, then the* $\mathfrak{I}_n$*graph has a super (8n+1,1)-edge-antimagic total labeling.*

Proof. Label the vertices of $\mathfrak{I}_n$ with $f_4(B_z) = f_1(B_z)$, $f_4(z_j) = f_1(z_j), f_4(x_i) = f_1(x_i)$ *and* $f_4(y_i) = f_1(y_i)$ if $1 \leq i \leq n, 1 \leq j \leq n+1$ and label the edges with the following way.

$$\begin{aligned}
f_4(Bz_1) &= 8n+3, \\
f_4(Bz_2) &= 12n+4, \\
f_4(Bz_3) &= 8n+1, \\
f_4(z_jz_{j+1}) &= 8n-2j+4, \text{untuk} 1 \leq j \leq 2n, \\
f_4(y_iz_{2i-1}) &= 12n-4i+6, \text{untuk} 1 \leq i \leq n, \\
f_4(x_iz_{2i}) &= 12n-4i+5, \text{untuk} 1 \leq i \leq n, \\
f_4(y_iz_{2i+1}) &= 12n-4i+4, \text{untuk} 1 \leq i \leq n, \\
f_4(x_iz_{2i+1}) &= 8n-4i+3, \text{untuk} 1 \leq i \leq n, \\
f_4(x_iz_{2i+2}) &= 12n-4i+3, \text{untuk} 1 \leq i \leq n-1, \\
f_4(x_iz_{2i+3}) &= 8n-4i+1, \text{untuk} 1 \leq i \leq n-1,
\end{aligned}$$

The total labeling f_4 is a bijective function from from $V\mathfrak{I}_n \cup E\mathfrak{I}_n$ onto the set$\{1,2,3,\ldots,4n+2\}$. The edge-weights of $\mathfrak{I}_n$, under the labeling f_4, constitute the sets.

$$\begin{aligned}
W_{f_4} &= \{w_{f_4} + f_4(Bz_1)\} = 8n+6 \\
W_{f_4} &= \{w_{f_4} + f_4(Bz_2)\} = 12n+7 \\
W_{f_4} &= \{w_{f_4} + f_4(Bz_3)\} = 8n+8 \\
W_{f_4} &= \{w_{f_4} + f_4(z_jz_{j+1}); \text{ jika } 1 \leq j \leq 2n\} = 8n+2j+5 \\
W_{f_4} &= \{w_{f4} + f_4(y_iz_{2i-1}); \text{ jika } 1 \leq i \leq n\} = 12n-4i+4 \\
W_{f_4} &= \{w_{f4} + f_4(x_iz_{2i}); \text{ jika } 1 \leq i \leq n\} = 12n+4i+5 \\
W_{f_4} &= \{w_{f4} + f_4(y_iz_{2i+1}); \text{ jika } 1 \leq i \leq n\} = 12n+4i+6 \\
W_{f_4} &= \{w_{f4} + f_4(x_iz_{2i+1}); \text{ jika } 1 \leq i \leq n\} = 8n+4i+6 \\
W_{f_4} &= \{w_{f4} + f_4(x_iz_{2i+2}); \text{ jika } 1 \leq i \leq n-1\} = 12n+4i+7 \\
W_{f_4} &= \{w_{f4} + f_4(x_iz_{2i+3}); \text{ jika } 1 \leq i \leq n\} = 8n+4i+8
\end{aligned}$$

It is not difficult to see that the set $w_{f_3} = \{8n+6, 8n+7, \ldots, 16n+6\}$ containsan arithmetic sequence with the first term $8n+6$ and common difference 1. Thus f_4 is a super $(8n+6;\ 1)$ $-$edge-antimagic total labeling. This concludes the proof.

CONCLUSION

Finally, we can conclude that the graph $\mathfrak{I}_n$admit a super (a, d) $-$edge antimagictotal labeling for all feasible d and $n \geq 2$.

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