

SUPER (a,d)-EDGE-ANTIMAGIC TOTAL LABELING OF SILKWORM GRAPH

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Abstract. An (a, d) -edge-antimagic total labeling of G is a one-to-one mapping f taking the vertices and edges onto $\{1, 2, 3, \dots, p + q\}$ such that the edge-weights $w(uv) = f(u) + f(v) + f(uv)$, $uv \in E(G)$ form an arithmetic sequence $\{a, a + d, a + 2d, \dots, a + (q - 1)d\}$, where first term $a > 0$ and common difference $d \geq 0$. Such a graph G is called *super* if the smallest possible labels appear on the vertices. In this paper we will study a super edge-antimagic total labelings properties of connective Sw_n graph. The result shows that a connected Silkworm graph admit a super (a, d) -edge antimagic total labeling for $d = 0, 1, 2$. It can be concluded that the result of this research has covered all the feasible n, d .

Key Words: (a, d) -edge-antimagic total labeling, super (a, d) -edge-antimagic total labeling, Silkworm graph.

INTRODUCTION

In mathematics and computer science, graph theory is used to model pairwise relations between objects from a certain collection. A "graph" in this context refers to a collection of vertices or 'nodes' and a collection of edges that connect pairs of vertices. A graph may be undirected, it means for two vertices u, v the edge $uv = \text{edge } vu$, or may be directed from one vertex to another. In this study we focus for undirected graph, and how to assign label on either vertex and edge.

A labeling of a graph is any mapping graph that sends some set of graph elements to a set of positive integers. If the domain is the vertex-set or the edge-set, the labelings are called, respectively, vertex labelings or edge labelings. Moreover, if the domain is $V(G) \cup E(G)$ then the labelings are called *total* labelings. We define the *edge-weight* of an edge $uv \in E(G)$ under a total labeling to be the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv . If such a labeling exists then G is said to be an (a, d) -edge-antimagic total graph. Such a graph G is called *super* if the smallest possible labels appear on the vertices. Thus, a *super (a, d) -edge-antimagic total graph* is a graph that admits a super (a, d) -edge-antimagic total labeling.

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This paper we investigate the existence of super (a, d) -edge-antimagic total labelings of Silkworm graph, and concentrate on the connected Silkworm graph denoted by Sw_n with vertex set $V(Sw_n) = \{x_i, y_i, z_j; 1 \leq i \leq n, 1 \leq j \leq n + 1\}$ and $E(Sw_n) = \{x_i z_i, x_i z_{i+1}, z_i z_{i+1}, y_i z_i, y_i z_{i+1}; 1 \leq i \leq n\} \cup \{y_i x_{i+1}; 1 \leq i \leq n - 1\}$. Thus $|V(Sw_n)| = p = 3n + 1$ and $|E(Sw_n)| = q = 6n - 1$.

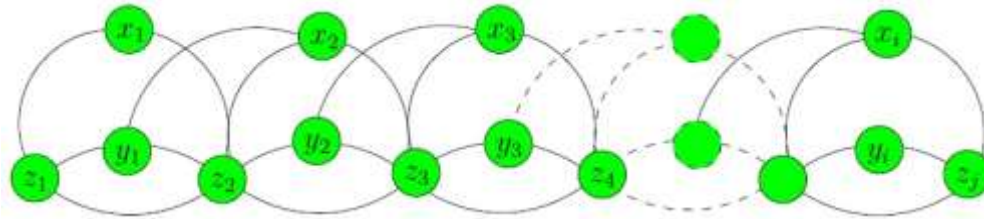


Figure 1: Silkworm graph Sw_n

RESEARCH METHODS

Research methods a super (a, d) -edge-antimagic total labeling of Silkworm graph are deductive axiomatic and the pattern recognition. The research techniques are as follows: (1) calculate the number of vertex p and size q on the graph Sw_n ; (2) determine the upper bound for values of d ; (3) determine the *EAVL* (edge-antimagic vertex labeling) of Sw_n ; (4) if the label of *EAVL* is expandable, then we continue to determine the bijective function of *EAVL*; (5) label the graph Sw_n with *SEATL* (super-edge antimagic total labeling) with feasible values of d and (6) determine the bijective function of super-edge. \square

Lemmas

We start this section by a necessary condition for a graph to be super (a, d) -edge antimagic total, providing a least upper bound for feasible values of d . This lemma can be found in [12]

Lemma 1. *If a (p, q) -graph is super (a, d) -edge antimagic total then $d \leq \frac{2p+q-5}{q-1}$*

Proof. Assume that a (p, q) -graph has a super (a, d) -edge antimagic total labeling $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, p + q\}$ and the edge-weights $\{a, a + d, a + 2d, \dots, a + (q - 1)d\}$. The minimum possible edge-weight in the labeling f is at least $1 + 2 + p + 1 = p + 4$. Thus, $a \geq p + 4$. On the other hand, the maximum

possible edge-weight is at most $(p - 1) + p + (p + q) = 3p + q - 1$. So we obtain $a + (q - 1) d \leq 3p + q - 1$ which gives the desired upper bound for the difference d . \square

Another important lemma obtained by Figueroa-Centeno et al [6], gives an easy way to find a total labeling for super edge-magicness of graph.

Lemma 2. A (p, q) -graph G is super edge-magic if and only if there exists a bijective function $f: V(G) \rightarrow \{1, 2, \dots, p\}$ such that the set $S = \{f(u) + f(v) : uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with magic constant $a = p + q + s$, where $s = \min(S)$ and $S = \{a - (p + 1), a - (p + 2), \dots, a - (p + q)\}$.

The two above lemma will be used for develop theorem 1.

RESULT AND DISCUSSIONS

If Silkworm graph has a super (a, d) -edge-antimagic total labeling then, for $p = 3n + 1$ and $q = 6n - 1$, it follows from Lemma 1 that the upper bound of d is $d \leq 2$ or $d \in \{0, 1, 2\}$. The following lemma describes an $(a, 1)$ -edge-antimagic vertex labeling for Silkworm graph.

Lemma 3 *If $n \geq 2$ then the Silkworm graph Sw_n has an $(3, 1)$ -edge-antimagic vertex labeling.*

Proof. Define the vertex labeling $f_1 : Sw_n \rightarrow \{1, 2, \dots, 3n + 1\}$ in the following way:

$$f_1(x_i) = 3i - 1, \text{ for } 1 \leq i \leq n$$

$$f_1(y_i) = 3i, \text{ for } 1 \leq i \leq n$$

$$f_1(z_j) = 3j - 2, \text{ for } 1 \leq j \leq n + 1$$

The vertex labeling is a bijective function. The edge-weights of Sw_n , under the labeling f_1 , constitute the following sets

$$wf_1(x_i z_i) = 6i - 3, \text{ for } 1 \leq i \leq n$$

$$wf_1(y_i z_i) = 6i - 2, \text{ for } 1 \leq i \leq n$$

$$wf_1(z_i z_{i+1}) = 6i - 1, \text{ for } 1 \leq i \leq n$$

$$wf_1(x_i z_{i+1}) = 6i, \text{ for } 1 \leq i \leq n$$

$$wf_1(y_i z_{i+1}) = 6i + 1, \text{ for } 1 \leq i \leq n$$

$$wf_1(y_i x_{i+1}) = 6i + 2, \text{ for } 1 \leq i \leq n - 1$$

It is not difficult to see that the set $wf_1 = \{3, 4, 5, \dots, 6n + 1\}$ consists of consecutive integers. Thus f_1 is a (3, 1)-edge antimagic vertex labeling. \square

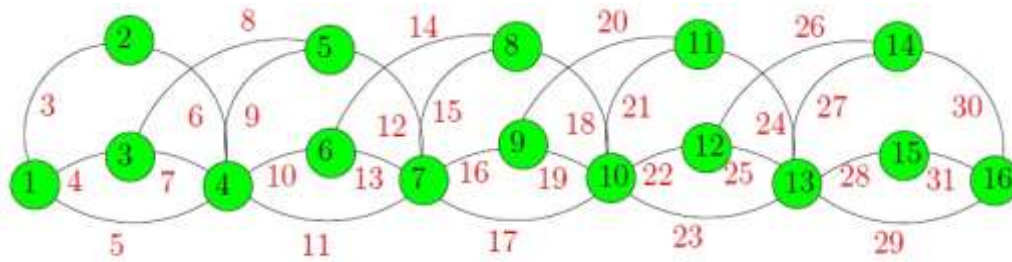


Figure 2: Vertex labeling (3,1)-edge antimagic base on Sw_5

Theorem 1 *If $n \geq 2$ then the graph Sw_n has a super $(9n + 3, 0)$ -edge-antimagic total labeling and a super $(3n + 5, 2)$ -edge-antimagic total labeling.*

Proof. *Case 1. for $d = 0$*

The edge label of Sw_n for $d = 0$ are:

$$f_2(x_i z_i) = 9n - 6i + 6, \text{ for } 1 \leq i \leq n$$

$$f_2(y_i z_i) = 9n - 6i + 5, \text{ for } 1 \leq i \leq n$$

$$f_2(z_i z_{i+1}) = 9n - 6i + 4, \text{ for } 1 \leq i \leq n$$

$$f_2(x_i z_{i+1}) = 9n - 6i + 3, \text{ for } 1 \leq i \leq n$$

$$f_2(y_i z_{i+1}) = 9n - 6i + 2, \text{ for } 1 \leq i \leq n$$

$$f_2(y_i x_{i+1}) = 9n - 6i + 1, \text{ for } 1 \leq i \leq n - 1$$

We have proved that the vertex labeling f_1 is a (3, 1)-edge antimagic vertex labeling. With respect to Lemma 1, by completing the edge labels $p + 1, p + 2, \dots, p + q$, we are able to extend labeling f_1 to a super $(a, 0)$ -edge-antimagic total labeling.

We can find the total labeling Wf_2 with summing $wf_1 = wf_2$ with edge label f_2 . It is not difficult to see that the set $Wf_2 = \{9n + 3, 9n + 3, \dots, 9n + 3\}$ contains an arithmetic sequence with the first term $9n + 3$ and common difference 0. Thus f_2 is a super $(9n + 3, 0)$ -edge-antimagic total labeling.

Proof. Case 2. For $d=2$

If $f_2(z)$ is edge label of Sw_n for $d=0$, and $f_3(z)$ is edge label of Sw_n for $d=2$, we can determine:

$$\begin{aligned} f_3(s) &= 2|p| + |q| + 1 - f_2(s) \\ &= 2(3n + 1) + (6n - 1) + 1 - f_2(s) \\ &= 12n + 2 - f_2(s) \end{aligned}$$

Then, if we substitute the edge label of $d=0$ to the formula, so we get the edge label of $d=2$:

$$\begin{aligned} f_3(x_i z_i) &= 3n + 6i - 4, \text{ for } 1 \leq i \leq n \\ f_3(y_i z_i) &= 3n + 6i - 3, \text{ for } 1 \leq i \leq n \\ f_3(z_i z_{i+1}) &= 3n + 6i - 2, \text{ for } 1 \leq i \leq n \\ f_3(x_i z_{i+1}) &= 3n + 6i - 1, \text{ for } 1 \leq i \leq n \\ f_3(y_i z_{i+1}) &= 3n + 6i, \text{ for } 1 \leq i \leq n \\ f_3(y_i x_{i+1}) &= 3n + 6i + 1, \text{ for } 1 \leq i \leq n - 1 \end{aligned}$$

The total labeling f_3 is a bijective function from $V(Sw_n) \cup E(Sw_n)$. The edge-weights of Sw_n , under the labeling f_2 . □

$$\begin{aligned} Wf_3(x_i z_i) &= 3n + 12i - 7, \text{ for } 1 \leq i \leq n \\ Wf_3(y_i z_i) &= 3n + 12i - 5, \text{ for } 1 \leq i \leq n \\ Wf_3(z_i z_{i+1}) &= 3n + 12i - 3, \text{ for } 1 \leq i \leq n \\ Wf_3(x_i z_{i+1}) &= 3n + 12i - 1, \text{ for } 1 \leq i \leq n \\ Wf_3(y_i z_{i+1}) &= 3n + 12i + 1, \text{ for } 1 \leq i \leq n \\ Wf_3(y_i x_{i+1}) &= 3n + 12i + 3, \text{ for } 1 \leq i \leq n - 1 \end{aligned}$$

From the first and second case we can conclude that if $n \geq 2$ then the graph Sw_n has a super $(9n + 3, 0)$ -edge-antimagic total labeling and a super $(3n + 5, 2)$ -edge antimagic total labeling, for $n \geq 2$.

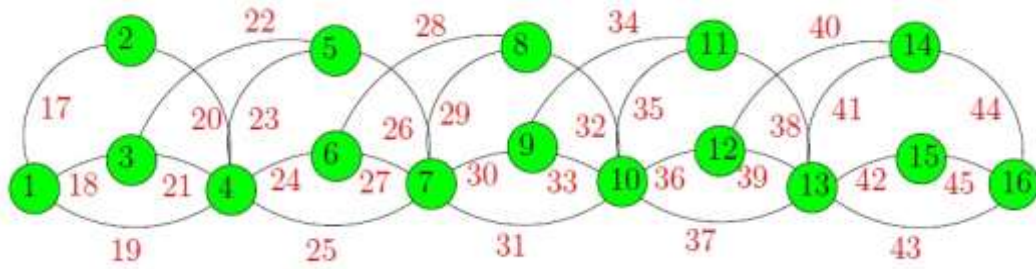


Figure 3: SEATL Silkworm graph Sw_5 for $d = 2$

Theorem 2 If $n \geq 2$ then the graph Sw_n has a super $(6n + 4, 1)$ -edge-antimagic total labeling.

Proof. let us define the vertex labeling of Silkworm graph Sw_n as $f_4(x_i z_i) = f_1(x_i z_i)$, $f_4(y_i z_i) = f_1(y_i z_i)$, $f_4(z_i z_{i+1}) = f_1(z_i z_{i+1})$, $f_4(x_i z_{i+1}) = f_1(x_i z_{i+1})$, $f_4(y_i z_{i+1}) = f_1(y_i z_{i+1})$, and $f_4(y_i x_{i+1}) = f_1(y_i x_{i+1})$ then the edge labeling f_4 for super $(a, 1)$ -edge total labeling can be defined as follow:

$$f_4(x_i z_i) = 6n - 3i + 4, \text{ for } 1 \leq i \leq n$$

$$f_4(y_i z_i) = 6n - 3i + 3, \text{ for } 1 \leq i \leq n$$

$$f_4(z_i z_{i+1}) = 6n - 3i + 2, \text{ for } 1 \leq i \leq n$$

$$f_4(x_i z_{i+1}) = 9n - 3i + 3, \text{ for } 1 \leq i \leq n$$

$$f_4(y_i z_{i+1}) = 9n - 3i + 2, \text{ for } 1 \leq i \leq n$$

$$f_4(y_i x_{i+1}) = 9n - 3i + 1, \text{ for } 1 \leq i \leq n - 1$$

The edge-weight of Sw_n under the labeling f_4 can be determine by adding the edge-weight of EAVL and the edge labeling by f_4 . Namely $Wf_4 = wf_1 + f_4$. We can have the following:

$$Wf_4(x_i z_i) = 6n + 3i + 1, \text{ for } 1 \leq i \leq n$$

$$Wf_4(y_i z_i) = 6n + 3i + 2, \text{ for } 1 \leq i \leq n$$

$$Wf_4(z_i z_{i+1}) = 6n + 3i + 3, \text{ for } 1 \leq i \leq n$$

$$Wf_4(x_i z_{i+1}) = 9n + 3i + 1, \text{ for } 1 \leq i \leq n$$

$$Wf_4(y_i z_{i+1}) = 9n + 3i + 2, \text{ for } 1 \leq i \leq n$$

$$Wf_4(y_i x_{i+1}) = 9n + 3i + 3, \text{ for } 1 \leq i \leq n - 1$$

We can state all the above edge-weights in a set $Wf_4 = \{6n + 4, 6n + 5, \dots, 12n + 2\}$. It can be seen that the edge-weights for a consecutive arithmetic sequence. The Silkworm graph Sw_n admit a super $(6n + 4, 1)$ -edge-antimagic total labeling. \square

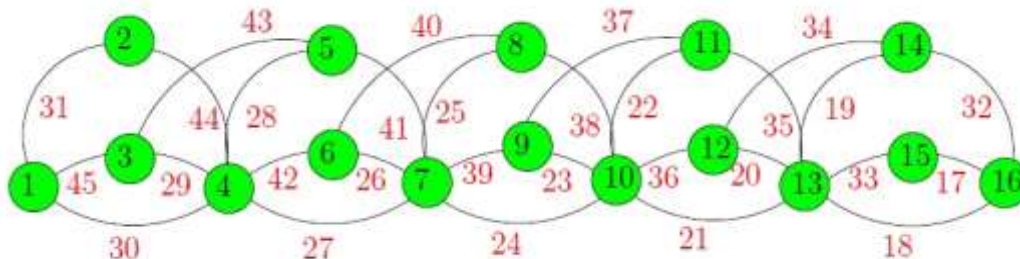


Figure 4: SEATL Silkworm graph Sw_5 for $d = 1$

CONCLUSION

Finally, we can conclude that the graph Sw_n admit a super (a,d) -edge antimagic total labeling for all feasible d and $n \geq 2$.

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