SUPER (a,d)-EDGE ANTIMAGIC TOTAL LABELING OF PENTAGONAL CHAIN GRAPH

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Abstract. *A G graph of order p and size q is called an (a,d)-edge antimagic total if there exist a bijection f: V(G)∪E(G) →{1,2,…,p+q} such that the edge-weights, w(uv)=f(u)+f(v)+f(uv), uv ∈ E(G), form an arithmetic sequence with first term a and common difference d. Such a graph G is called super if the smallest possible labels appear on the vertices. In this paper we study super (a, d)-edge-antimagic total properties of connected PCn by using deductive axiomatic and the pattern recognition method. The result shows that a connected pentagonal chain graphs admit a super (a,d)-edge antimagic total labeling for d = 0,1,2 for n* ≥ 1. *It can be concluded that the result of this research has covered all the feasible d.*

Key Words: *(a,d)-edge antimagic vertex labeling, super (a,d)-edge antimagic total labeling, Pentagonal Chain Graph.*

INTRODUCTION

Mathematics is the basic of all knowledge in the world which be related tonumbers, logic, etc. Mathematics consists of some knowledges, such as: economic mathematics, pure mathematics, discrete mathematics, etc. One of theory in discrete mathematics is graph theory, which has super edge antimagic total labeling (SEATL). In this research will be investigated super edge antimagic total labeling on Pentagonal Chain Graph join and disjoin. Pentagonal Chain Graph join is defined by labelling one or more graph which join in that graph. For example a graph in Pentagonal chain is defined a pentagon. But when two graph is two pentagon with chain. It's different with disjoin graph which the labelling is organized between three or more chain graph. How to labelling this graph is easy. It's started from x_1 for label 1, then $x_{1,3}$ for label 2, after that $x_{1,1}$ for label 3, then $x_{1,2}$ for label 4, and then x_2 for label 5, then go on for next label number. For the disjpoin graph, labeling follows the join graph for the series.

By a *labeling* we mean any mapping that carries a set of graph elements onto a set of numbers, called *labels*. In this paper, we deal with labelings with domain the set of all vertices and edges. This type of labeling belongs to the class of *total* labelings. We define the *edge-weight* of an edge *uv* ∈ *E(G)* under a total labeling to be the sum of the vertex labels corresponding to vertices *u*, *v* and edge label corresponding to edge *uv*. In

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this paper we investigate the existence of super *(a,d)*-edge-antimagic total labelings for connected and disconnected graphs. Some constructions of super *(a,d)*-edge-antimagic total labelings for $m\mathcal{L}_n$ and $m\mathcal{L}_{\{i,j,k\}}$ have been shown by Dafik, Slamin, Fuad and Rahmad in [3] and super (a,d)-edge-antimagic total labelings for Generalized Petersen (n,*2*) have been described by Debby.

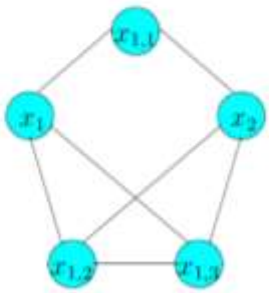

Figure 1: Super (a,d)-edge antimagic total labeling of $\mathfrak{PC}_1$

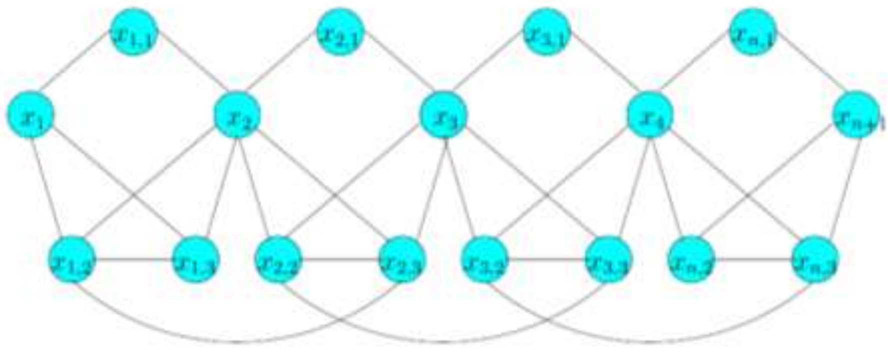

Figure 2: Super (a,d)-edge antimagic total labeling of $\mathfrak{PC}_n$

Dafik *et al also* found some families of graph which admits super (a,d)-edge-antimagic total labelings, namely mC_n, mP_n, $mK_{n,n,.,n}$.

We will now concentrate on the connected Pentagonal Chain, denoted by PC_n. Pentagonal Chain graph has vertexes such that $VPC_n = \{x_i\ x_{ij}; 1 \leq i \leq n; 1 \leq j \leq 3\}$ and edges such that $EPC_n = \{x_i\ x_{ij}; x_{ij}x_{i+1}; 1 \leq i \leq n; 1 \leq j \leq 3\}$. So that $|V(PC_n)| = p = 4n+1$ and $|E(PC_n)| = q = 8n-1$.

Super *(a,d)*-edge Antimagic Total Labeling

An *(a,d)*-edge-antimagic total labeling on a graph G is a bijective function $f{:}V(G) \cup E(G) \rightarrow \{1,2,\ldots,p+q\}$\$ with the property that the edge-weights $w(uv)=f(u)+f(uv)+f(v)$, $uv \in E(G)$, form an arithmetic progression $\{a,a+d,a+2d,\ldots,a+(q-1)d\}$. where $a>0$ and $d \geq 0$ are two fixed integers. If such a labeling exists then G is said to be an *(a,d)*-edge-antimagic total graph. Such a graph G is called super if the smallest possible labels appear on the vertices. Thus, a super *(a,d)*-edge-antimagic total graph is a graph that admits a super *(a,d)*-edge-antimagic total labeling. We continue this section by a necessary condition for a graph to be super *(a,d)*-edge-antimagic total, providing a least upper bound for feasible values of *d*

Lemma 1 *If a (p,q)-graph is super (a,d)-edge-antimagic total then* $d \leq \frac{2p+q-5}{q-1}$.

Proof. Assume that a *(p,q)*-graph has a super *(a,d)*-edge-antimagic total labeling $f: V(G) \cup E(G) \to \{ 1,2,..., p+q \}$. The minimum possible edge-weight in the labeling *f* is at least 1+ 2 + p+1 = p+4. Thus, a ≥ p+4. On the other hand, the maximum possible edge-weight is at most (p-1) + p + (p+q) = 3p +q - 1. So we obtain a + (q-1)d ≤ 3p + q - 1 which gives the desired upper bound for the difference *d*

Lemma 2 A *(p,q)*-graph G is super edge-magic if and only if there exists a bijective function f :V(G)→{ 1,2,…, p } such that the set S={ $f(u)+f(v): uv \in E(G)$} consists of q consecutive integers. In such a case, *f* extends to a super edge-magic labeling of G with magic constant a = p + q + s, where s = min(S) and S= { a- (p+1), a- (p+2), ….,a-(p+q)}

In our terminology, the previous lemma states that a *(p,q)*-graph G is super *(a,0)*-edge-antimagic total if and only if there exists an (a – p - q, 1)-edge-antimagic vertex labeling.

RESEARCH METHODS

In this paper, we using a pattern recognition and axiomatic deductive to get the bijection function of super (a,d)-edge antimagic total labeling of Pentagonal Chain graph. The research techniques are as follows: (1) calculate the number of vertex p and size q of graph PC_n and mPC_n; (2) determine the upper bound for values of d; (3) determine the label of *EAVL* (edge-antimagic vertex labeling) of PC_n and $sR_{m,n}$; (4) if the label of *EAVL* is expandable, then we continue to determine the bijective function of *EAVL*; (5) label the graph PC_n and mPC_n with *SEATL* (super-edge antimagic total labeling) with feasible values of d by using Lemma 1 and (6) determine the bijective function of super-edge antimagic total labeling of graph PC_n and mPC_n.

RESULT AND DISCUSSIONS

If Pentagonal Chain graph, has a super (a,d)-edge-antimagic total labeling then, for p = 4n + 1 and q = 8n - 1, it follows from Lemma 1 that the upper bound of d is d ≤ 2 or d ∈ {0,1,2}. The following lemma describes an (a,1)-edge-antimagic vertex labeling for Pentagonal Chain.

Lemma 3 *If 1 ≤ i ≤ n then the Pentagonal Chain graph PCn has an (8i - 5, 1)-edge-antimagic vertex labeling.*

Proof. Define the vertex labeling:

$f_1(x_i) = 4i - 3, if\ 1 \le i \le n$

$f_1(x_{i3}) = 4i - 2, if\ 1 \le i \le n$

$f_1(x_{i1}) = 4i - 1, if\ 1 \le i \le n$

$f_1(x_{i2}) = 4i, if\ 1 \le i \le n$

The vertex labeling f_1 is a bijective function. The edge-weights of PC_n, under the labeling f_1 , constitute the following sets:

$w_{f_1}^1(x_i x_{i3}) = 8i - 5; if\ 1 \le i \le n$

$w_{f_1}^2(x_i x_{i1}) = 8i - 4; if\ 1 \le i \le n$

$w_{f_1}^3(x_i x_{i2}) = 8i - 3; if\ 1 \le i \le n$

$w_{f_1}^4(x_{i2} x_{i3}) = 8i - 2; if\ 1 \le i \le n$

$w_{f_1}^5(x_{i3} x_{i+1}) = 8i - 1; if\ 1 \le i \le n$

$w_{f_1}^6(x_{i1} x_{i+1}) = 8i; if\ 1 \le i \le n$

$w_{f_1}^7(x_{i2} x_{i+1}) = 8i + 1; if\ 1 \le i \le n$

$w_{f_1}^8\left(x_{i2} x_{i+1,3}\right) = 8i + 2; if\ 1 \le i \le n - 1$

It is not difficult to see that the set $U_{t=1}^{13} W_{f_1}^{t}$={3,4,…,8i+1} consists of consecutive integers. Thus f_1 is a (8i-5,1)-edge antimagic vertex labeling. Baca, Y. Lin, M. Miller and R. Simanjuntak [13], Theorem 5) have proved that if *(p,q)*-graph G has an *(a,d)*-edge antimagic vertex labeling then G has a super *(a+p+q,d-1)*-edge antimagic total labeling and a super *(a+p+1,d+1)*-edge antimagic total labeling. With the Lemma 3 in hand, and using Theorem 5 from [13], we obtain the following result.

Theorem 1 *If m ≥ 2 and n ≥ 1 then the graph PCn has a super (12n+3,0)-edge-antimagic total labeling and a super (12n-35,2)-edge-antimagic total labeling.*

Proof.

Case 1. d=0

Label the vertices of PC_n with:

$w^1_{f_2}(x_{i2}x_{i+1,3}) = 12n - 8i + 1; if\ 1 \le i \le n-1$

$w^2_{f_2}(x_{i2}x_{i+1}) = 12n - 8i + 2; if\ 1 \le i \le n$

$w^3_{f_2}(x_{i1}x_{i+1}) = 12n - 8i + 3; if\ 1 \le i \le n$

$w^4_{f_2}(x_{i3}x_{i+1}) = 12n - 8i + 4; if\ 1 \le i \le n$

$w^5_{f_2}(x_{i2}x_{i3}) = 12n - 8i + 5; if\ 1 \le i \le n$

$w^6_{f_2}(x_{i}x_{i2}) = 12n - 8i + 6; if\ 1 \le i \le n$

$w^7_{f_2}(x_{i}x_{i1}) = 12n - 8i + 7; if\ 1 \le i \le n$

$w^8_{f_2}(x_{i}x_{i3}) = 12n - 8i + 8; if\ 1 \le i \le n$

It follows from Lemma 2 that the labeling f_2 can be extended, by completing the edge label p+1, p+2,… , p+q, to a super (a,0)-edge antimagic total labeling, where, in the case p = 4n + 1 and q = 8n - 1.

We can found the total labeling W_{f_2} with summing $w_{f1} = w_{f2}$ with edge label f_2. It is not difficult to see that the set $U^{13}_{t=1} W^t_{f_2}$ = {12n + 3, 12n + 3,…, 12n + 3} contains an arithmetic sequence with the first term 6m+6n+9 and common difference 0. Thus f_2 is a super (12n + 3,0)-edge-antimagic total labeling. This concludes the proof.

Case 2, d=2.

Let $f_2(0)$ be an edge label of PC_n for d=0. Based on the edge label position under the edge weight of EAVL w then the edge label $f_3(0)$ for d=2 can be formulated as follows:

$$\begin{aligned} f_3(u) &= 2|v| + |e| + 1 - f_2(u) \\ &= 2(4n+1) + (8n-1) + 1 - f_2(u) \\ &= 16n + 1 - f_2(u) \end{aligned}$$

The total labeling f_3 is bijective function from $V(PC_n) \cup E(PC_n)$. The edge weight of PC_n, under the labeling f_2, constitute sets

$$W^1_{f_3}(x_i x_{i3}) = 12n + 16i - 51; jika\ 1 \le i \le n$$

$$W^2_{f_3}(x_i x_{i1}) = 12n + 16i - 49; jika\ 1 \le i \le n$$

$$W^3_{f_3}(x_i x_{i2}) = 12n + 16i - 47; jika\ 1 \le i \le n$$

$$W_{f_3}^{4}(x_{i2}x_{i3}) = 12n + 16i - 45; jika\ 1 \leq i \leq n$$
$$W_{f_3}^{5}(x_{i3}x_{i+1}) = 12n + 16i - 43; jika\ 1 \leq i \leq n$$
$$W_{f_3}^{6}(x_{i1}x_{i+1}) = 12n + 16i - 41; jika\ 1 \leq i \leq n$$
$$W_{f_3}^{7}(x_{i2}x_{i+1,3}) = 12n + 16i - 37; jika\ 1 \leq i \leq n - 1$$
$$W_{f_3}^{8}(x_{i2}x_{i+1}) = 12n + 16i - 39; jika\ 1 \leq i \leq n$$

It is not difficult to see that the set $U_{t=1}^{8} W_{f_3}^{t}$= {12n - 35, 12n - 33, 12n - 31, ..., 28n - 39} contains an arithmetic sequence with a= 12n - 35 and d=2. Thus f_3 is a super (12n - 35,2)-edge-antimagic total labeling. This concludes the proof.

Lemma 4 Let ψ is a set of constitutif of integer $\psi = \{c, c+1, c+2, \dots, c+k\}\ with\ k\ is\ even.$ So, there is permutation $\mathrm{II}(\psi)$ from elemen of the set ψ so $\psi + \mathrm{II}(\psi)$ is a set constitutive integer too $\psi + \mathrm{II}(\psi) = \left\{2c + \frac{k}{2}, 2c + \frac{k}{2} + 1, 2c + \frac{k}{2} + 2, \dots, 2c + \frac{3k}{2}\right\}$.

Proof.

Let ψ is a set of constitutive integers $\psi = \{v_i | v_i = c + (i-1), 1 \leq i \leq k+1\}$ and k is even. Define of value of permutation $\mathrm{II}(\psi) = \{w_i | 1 \leq i \leq k+1\}$ from the elemen ψ is

$$w_i = \begin{cases} c+i+\frac{k}{2}, if\ 1 \leq i \leq \frac{k}{2} \\ c+i-\left(\frac{k}{2}+1\right), if\ \frac{k}{2}+1 \leq i \leq k+1 \end{cases}$$

Pentagonal Chain graph has labeling (8n-5, 1)-EAV. It means that graph PC_n have set of edge weights by vertex labeling f_1 stated in {8n-5, 8n-4, ..., 8n+1}, or PC_n having a row edge weights with initial value a = 8n-5 and d=1. If let row of edge weight PC_n expressed in $\Upsilon = \{c, c+1, c+2, \dots, c+k\}$ so we get the value of c = 8n-5 and k = 8n-12. $\mathrm{II}(\Upsilon)$ is permutation value Υ so value of $\Upsilon + (\mathrm{II}(\Upsilon) + \frac{\mathrm{k}}{2} - 1)$ is total weight from the function. Let formula that used is i even i=2 is the smallest weight so:

$$\Upsilon + (\mathrm{II}(\Upsilon) + \eta) = a$$
$$c + 1 + \left(c + \frac{k}{2} + \frac{2-i}{2}\right) + \eta = 12n - 16$$
$$2c + 1 + \frac{k}{2} + \frac{2-2}{2} + \eta = 12n - 16$$
$$2(8n-5) + 1 + \frac{8n-12}{2} + 0 + \eta = 12n - 16$$
$$\eta = 12n - 20n - 15 + 16$$

$$\eta = -8n - 1$$
$$\eta = -2c + k + 1$$

If we substitute the value of c = 8n-5 and k = 8n-12 so we get

$$\Upsilon + (\text{II}(\Upsilon) + \eta) = c + (i-1) + (c + \frac{k}{2} + \frac{2-i}{2} - 2c + k + 1)$$
$$= \frac{3k}{2} + (i-1) + \frac{2-i}{2} + 1$$

Theorem 2 If m ≥ 2 and n ≥ 2, then the graph PC*n* has a super (12n-16,1)-edge-antimagic total labeling.

Proof. Label the vertices of PC_n with $f_4(x_i x_{ij}) = f_1(x_i x_{ij}), f_4(x_{i2}x_{i3}) = f_1(x_{i2}x_{i3}), f_4(x_{ij}x_{i+1}) = f_1(x_{ij}x_{i+1}), f_4(x_{i2}x_{i+1,3}) = f_1(x_{i2}x_{i+1,3}), for\ 1 \le i \le n, 1 \le j \le 3$ and label the edges with the following way.

$f_4(x_{i2}x_{i+1}) = 12n - 4i - 18; jika\ 1 \le i \le n$

$f_4(x_{i3}x_{i+1}) = 12n - 4i - 17; jika\ 1 \le i \le n$

$f_4(x_i x_{i2}) = 12n - 4i - 16; jika\ 1 \le i \le n$

$f_4(x_i x_{i3}) = 12n - 4i - 15; jika\ 1 \le i \le n$

$f_4(x_{i2}x_{i+1,3}) = 12n - 4i + 1; jika\ 1 \le i \le n-1$

$f_4(x_{i1}x_{i+1}) = 12n - 4i + 2; jika\ 1 \le i \le n$

$f_4(x_{i2}x_{i3}) = 12n - 4i + 3; jika\ 1 \le i \le n$

$f_4(x_i x_{i1}) = 12n - 4i + 4; jika\ 1 \le i \le n$

If w_{f_4} defines as total labeling edge weight of labeling f_4 then w_{f_4} can get by sum formula edge weight EAVL $w_{f_4} = w_{f_1}$ and formula of edge weight f_4 with limit i such that:

$W_{f_4}^1(x_i x_{i3}) = 12n + 4i - 20; jika\ 1 \le i \le n$

$W_{f_4}^2(x_i x_{i2}) = 12n + 4i - 19; jika\ 1 \le i \le n$

$W_{f_4}^3(x_{i3}x_{i+1}) = 12n + 4i - 18; jika\ 1 \le i \le n$

$W_{f_4}^4(x_{i2}x_{i+1}) = 12n + 4i - 17; jika\ 1 \le i \le n$

$W_{f_4}^5(x_i x_{i1}) = 12n + 4i; jika\ 1 \le i \le n$

$W_{f_4}^6(x_{i2}x_{i3}) = 12n + 4i + 1; jika\ 1 \le i \le n$

$W_{f_4}^7(x_{i2}x_{i+1,3}) = 12n + 4i + 2; jika\ 1 \le i \le n-1$

$W_{f_4}^8(x_{i1}x_{i+1}) = 12n + 4i + 3; jika\ 1 \le i \le n$

CONCLUSION

Based on the results of the discussion, can be conclude that:

There are a super (a,d)-edge-antimagic total labeling of graph PC_n , if $n \leq 1$ with $d \in \{0, 1, 2\}$.

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