THE EFFECTIVENESS OF ADAMSBASHFORTH-MOULTONORDER 12 METHOD IN ANALYZING THE RABIES VIRUS TRANSMISSION MODEL

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Abstract. *Tujuan dari penelitian ini adalah untuk mengetahui efektivitas metode Adams Bashforth-Moulton dalam menganalisis model penularan virus Rabies. Model matematika sistem penularan virus Rabies direpresentasikan dalam bentuk persamaan diferensial biasa (PDB) non linear orde satu sehingga sulit diselesaikan dengan metode analitik. Metode numerik Adams Bashforth-Moulton order dua belas digunakan dalam penelitian ini karena sudah terbukti merupakan metode yang lebih teliti dalam menyelesaikan permasalahan yang sulit diselesaikan secara analitik. Sebagai perbandingan, digunakan metode Adams Bashforth-Moulton order sembilan untuk menganalisis tingkat keakuratan dan keefektivannya. Hasil penelitian menunjukkan bahwa metode Adams Bashforth-Moulton order dua belas memiliki nilai error yang lebih kecil dibandingkan metode Adams Bashforth-Moulton order sembilan. Hal ini menunjukkan metode Adams Bashforth-Moulton order dua belas lebih efektif dalam menganalisis model dinamika penularan virus Rabies.*

Kata Kunci: *Model Dinamika Penularan Virus Rabies, Metode Adams Bashforth-Moulton Order 12, program MATLAB*

INTRODUCTION

A problem in the real world is often difficult to be analyzed, so we need anapproximation to explain, predict or determine the answer of the problem beinginvestigated. One of approximation to explain the solution of the problemoccurred in the real world is to translate the problem into mathematical language.By translating the problem into mathematical language we will get themathematical models of the problem, namely the formulation results which areillustrate the problem that the answer will be searched.

Mathematical models can be applied to analyze the transmission dynamics of*Rabies* virus. *Rabies* is an acute infectious disease of the nervous system centercaused by *Rabies* virus. Rabies disease transmission most caused by the bite ofan animal, especially dogs, cats, and monkeys (on the other cases are caused bybat bites). Rabies is a disease *zoonotic*, meaning that it can be easily transmittedfrom animals to humans and can be fatal if not promptly treated.

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Adams Bashforth-Moulton methods is one of numerical methods that can be used to obtain the solutions of differential equations with initial value conditions are known. This method was developed based on a Taylor series expansion beheading its terms at a higher tier. So, the Adams Bashforth-Moulton order 12 method can be used to search the solution of ordinary differential equation model case of rabies virus transmission.

Differential equation is an equation consists of one or more dependent variablesto one or more independent variables. If the derivative of the functionis only depend on the independent variable so it's called Ordinary DifferentialEquations (ODE), and when depends on more than one independent variableso it's called Partial Differential Equations (PDE). In this study, the diseasemodel used is the mathematical model of *rabies* virus transmission systemwhich is type of non-linear first order of ODE.

In this research, the mathematic model of Rabies virus transmission that willbe analyzed is *SEIR* model. SEIR model is a model that classifies the compartmentdog population into four classes, namely class *susceptible* (S), grade*exposed* (E), *infectious* class (I) and *recovered* class (R), developed by Kwaku MariAddo. Then we used Adams Bashforth-Moulton order 12 to analyze the modeland using MATLAB as a software helper. "Differential equations with AdamsBashforth-Moulton solution can be sought in the form of tables and graphswhen the input data and variables used must be adjusted prior to the variablesused in the computer program in order to facilitate the view settlementproceeds and any programming language that can be used is a computing languageMATLAB (Matrix Laboratory)"(Ayuninghemi and Kapakisan, 2009:1).

In this research, the researchers also examined the level effectivity of AdamsBashforth-Moulton order 12 by using the comparative method of Adams Bashforth-Moulton order 9.Rabies virus transmission dynamics model of the dogs were divided into twodifferent treatments (with vaccination and without vaccination). The systemsgoverned by first order of non-linear ordinary differential equations as follow:

$$\frac{dS}{dt} = \delta + \kappa R - \beta SI - \alpha S - \mu S \qquad (1)$$

$$\frac{dE}{dt} = \beta SI - \alpha E - \mu E - \nu E \qquad (2)$$

$$\frac{dI}{dt} = \nu E - \mu I - \epsilon I \quad (3)$$

$$\frac{dR}{dt} = \alpha S + \alpha E - \mu S - \kappa R \quad (4)$$

$$\frac{dS}{dt} = \delta + \kappa R - \beta SI - \alpha S - \mu S \quad (5)$$

$$\frac{dE}{dt} = \beta SI - \alpha E - \mu E - \nu E \quad (6)$$

$$\frac{dI}{dt} = \nu E - \mu I - \epsilon I \quad (7)$$

$$\frac{dR}{dt} = \alpha S + \alpha E - \mu S - \kappa R \quad (8)$$

Equation (1) to (4) is a *Rabies* virus transmission equation model in dogs with vaccination, while equation (5) to (6) is the *Rabies* virus transmission equation model without giving vaccination (non-vaccinated). The Interpretation of the parameters of the equations taken based on the results of research that addresses the numerical simulation *Rabies* virus epidemic that is written in a case study report named "*An SEIR Mathematical Model for Dog Rabies. Case Study: Bongo District, Ghana*" by Kwaku Mari Addo. The initial and the estimated value of each parameters used are taken from the *Ghana Veterinary Medical Association Report 2010* in the similar reports, as shown in the following table:

Table 1. Symbols and Definitions of Sub-Population Dogs

Symbol	Name	Definition
S	*Susceptible*	Uninfected but susceptible to infection
E	*Exposed*	Infected but can not infect other dogs (latent or carrier)
I	*Infectious*	Rabies infection is active (can infect other dogs)
R	*Recovered*	Healed (from the Rabies-latent infection or active-Rabies)

The initial value of each sub population of dogs (with vaccination in dogs) are: (0)=399, E(0)=0, I(0)=1, and R(0)=100 and the initial value of each sub population of dogs (without vaccination) are: S(0)=499, E(0)=0, I(0)=1, and R(0)=0.

Table 2. The Interpretation of Parameter

Parameter	Description	The Estimation Value
β	transmission coefficient between dogs	3,0417 x 10^{-3}

ν	the incubation period in dogs	2,1429 x 10^{-3}
γ	mortality rate in dogs (without vaccination)	2,293 x 10^{-3}
δ	birth rate in dogs	0.1975 x 10^{-3}
ϵ	disease induced mortality rate	4,9167 x 10^{-3}
κ	wanning Immunity Rate in dogs	1,9177x 10^{-3}
α	vaccination rates in dogs	2,975 x 10^{-3}
μ	mortality rate in dogs (with vaccination)	2,293x 10^{-3}

Mathematically, the modeling of *Rabies* virus transmission system showed the first non-linear order of an ordinary differential equations so it is difficult to be solved analytically. Therefore,the methods that can be used to analyze the *Rabies* virus transmission system is by numerical methods. Adam Bashforth-Moulton methods are included in the Linear Multistep methods which is a simple method of numerical resolution, because it doesn't need to find the derivatives of functions, but only the equation of predictor and corrector. In this case, Adams Bashforth method as a predictor and Adams Moulton method as a corrector. The advantage of this method gives solution that is accurate and stable both exact solution and the approximation (Lestari, 2011:6).

According to Shampine (1994:188-189), The general method of Multistep canbe written as follow:

$$\sum_{i=0}^{k} \alpha_i \, y_{n+1} = h \sum_{i=0}^{k} \beta_i \, f_{n+1}$$

If the value of $\beta_k = 0$ so this method called explicit multistep method, and if the value of$\beta_k \neq 0$ so that this method called implicit multistep method. Then,this method can be classified into two form of polinomial, denoted by ρ and σ.

$$\rho(s) = \alpha_k \, s^k + \alpha_{k-1} \, s^{k-1} + \ldots + \alpha_0 (left\ side) \quad (10)$$

and

$$\sigma(s) = \beta_k \, s^k + \beta_{k-1} \, s^{k-1} + \ldots + \beta_0 (right\ side) \quad (11)$$

Predictor-corrector method is a set of two equations for y_{n+1}. This first equation called predictor and used to predict (get the first approximation for y_{n+1}. The second equation called corrector, and used to get the correct value (second approximation for y_{n+1}). Generally, the corrector is depend on the value predicted.

Predictor is called the explicit formula because it's provide the explicit way in calculating y_n. and $hy'_n$from the *y* value and derivatif on the point before. Correctoris called the implicit formula because in general is a nonlinear equationinvolving *f* function and must be solved to y_n(William, 1971:105-112). The correctorformula is more accurate than the predictor one, although both of themhave a same of local *error* value. It is because the coefficient in the *error* rate isless than the predictor one. (Conte and Carl, 1993:344).

Generally, The Adams Bashforth-Moulton Method formula as follow:

$$y_n = \sum_{i=1}^{k} \alpha_i \, y_{n-i} + \beta_i \, hf_{n-i}$$

Where $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = \alpha_4 = \cdots = \alpha_k = 0$.For the Adams Bashforth order 12, the *n* value is 12, and for the Adams Moulton order 12, the *n* value is 11. For a method of approaching the actual solution, the method should be convergent. However, Dafik (1999:83) explained that a consistent method, can't be readily ascertained automatically that the method also converges. Therefore, the Another necessary condition is zero-stable.

Definition 1.(**ZERO-STABLE**) *A method is said to have the properties of zero-stable or meet the conditions of the roots when the roots of* $\rho(s) = 0$ *meet nature*$|S_n| \leq 1$. *WhenallS*$_n$= 1 *then that method is said to be stable.*

Theorem 1. *If the Linear Multistep methods (MML) consistently and simultaneouslysatisfies the zero-stable, then the method is said to be convergent.*

consistent + zero stable → convergent

Definition 2. (**CONVERGENCE**) A method said to be convergent if

$$\max_{0 \leq i \leq n} \| y(x_i) - y_i \| \to 0 \quad untuk \quad h \to 0 \qquad (13)$$

Note:

l_n = local beheading error

$y(x_{n+1})$ = analitical or eksak solution

y_{n+1} = numerical or approximation solution

RESEARCH METHODS

In this section will be explained the research methods that includes that consist of research procedures, methods of data collection and analysis data. In accordance with the steps in solving mathematical modeling with numerical methods, the research procedure will be done as follows:

1. make the reduction formulas of Adams Bashforth-Moulton order 12 method theoretically;
2. determine The convergence of Adams Bashforth-Moulton order 12 method theoretically;
3. (modeling phase) using mathematical modeling based on referral the researchers took from the research journal which is shaped first order of ordinarydifferential equations (ODE) system;
4. (numerical formulation phase) formulate the ordinary differential equations system of Rabies Transmission virus model numerically;
5. (algorithms phase) make the pattern algorithms of Adams Bashforth-Moulton order 12 method;
6. (programming phase) make listing program of Adams Bashforth-Moulton order 12 method in MATLAB;
7. collect the secondary data to determine the effectiveness of the numericalsolution and Adam Bashforth-Moulton order 12 method;
8. enter the data that has been obtained into the format programming;
9. (operational phase) determine the numerical solution and collect the dataconvergence of the execution of the Rabies transmission virus model;
10. (the evaluation phase) analyzed the data obtained;
11. make conclusions.

For the method of data collection, the researcher used an experimental anddocumentation method. It is because the researcher make observations andrecords held by the systematic review of factors, data and symptoms of whatwould happen. Things were observed in this research are the value of error andthe convergence graphs. By using that observational data, we can know whatmethod is more effective. Especially for the simulation case, the researcher useda documentation method by collecting every variable consist of notes, agendas,and other connected to the research. This research used secondarydata because the data used obtained from the research journals, namely "*AnSeir Mathematical Model for Dog Rabies. Case Study: Bongo District, Ghana*" writtenby Kwaku Mari Addo. The data contains of the location of

Rabies diseaseand population, and the number of each sub-populations that affect patterns ofRabies disease spreading.

In analyzing the data, the researcher used the descriptive method for presenteddata tables and graphs from the execution results. The data will be taken is theerror value of both methods at the same iteration and the convergence graphs.
The convergence can be seen from the small errors generated at each iterationor near zero. The method that have smaller error called more effective thenones and vice versa.

RESEARCH FINDINGS AND DISCUSSIONS

In this section will be discussed the results of this research accordance to theformulation of the problem. The formulation of Adams Bashforth method as apredictor, is

$$y_{n+12} = y_{n+11} + \frac{h}{958003200}(4527766399f_{n+11} - 19433810163f_{n+10} \\ +61633227185f_{n+9} - 135579356757f_{n+8} + 214139355366f_{n+7} \\ -247741639374f_{n+6} + 211103573298f_{n+5} - 131365867290f_{n+4} \\ +58189107627f_{n+3} - 17410248271f_{n+2} + 3158642445f_{n+1} \\ -262747265f_n)$$

The formulation of Adams Moulton method as a corrector is:

$$y_{n+12} = y_{n+11} + \frac{h}{958003200}(262747265f_{n+12} + 1374799219f_{n+11} \\ -2092490673f_{n+10} + 3828828885f_{n+9} - 5519460582f_{n+8} \\ +6043521486f_{n+7} - 4963166514f_{n+6} + 3007739418f_{n+5} \\ -1305971115f_{n+4} + 384709327f_{n+3} - 68928781f_{n+2} \\ +5675265f_{n+1}$$

From the result of the research, the formula of Adams Bashforth-Moulton order12 satisfied consistent and zero stable, so according to the theorem 1, the Adams Bashforth-Moulton order 12 is a convergent method.Then, after the formula of Adams Bashforth-Moulton and the mathematic modelhave been converted to the MATLAB language, and has been runnning usingiteration 250000, we get the graph convergence as follow:

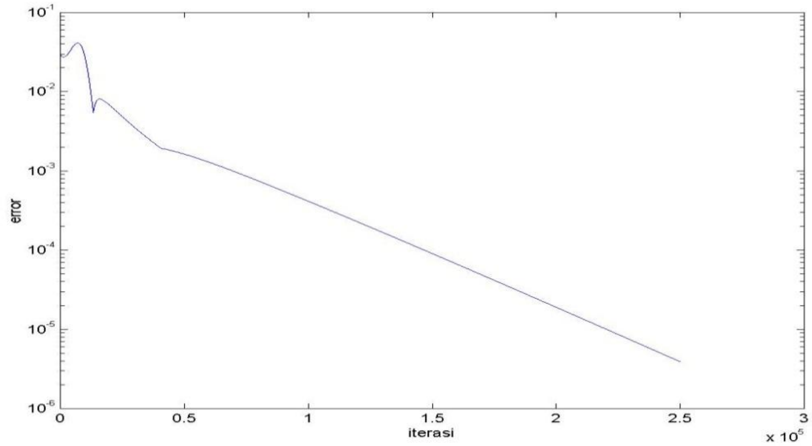

Figure 1: Graph Error of ABM 12 on the *Rabies* Virus Transmisson Model (With Vaccination) at Iteration 250000.

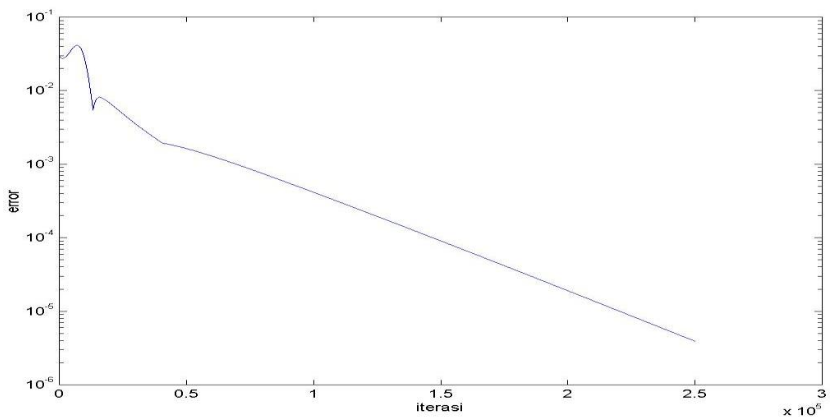

Figure 2: Graph Error of ABM 9 on the *Rabies* Virus Transmisson Model (With Vaccination) at Iteration 250000

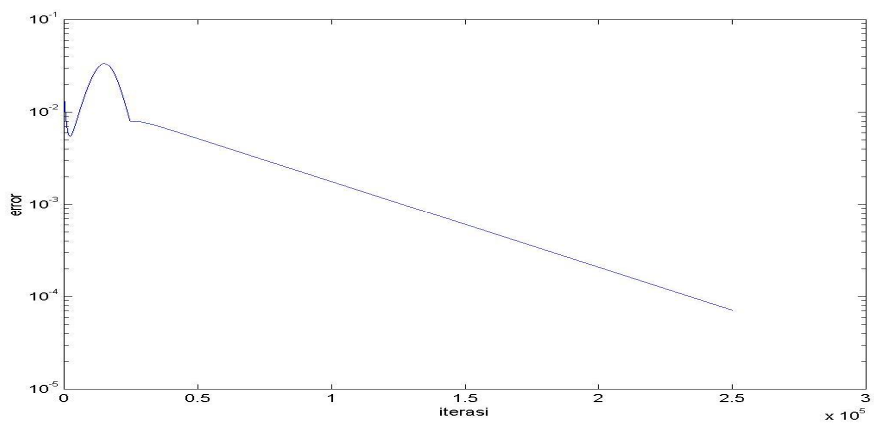

Figure 3: Graph Error of ABM 12 on the *Rabies* Virus Transmisson Model (without Vaccination) at Iteration 250000

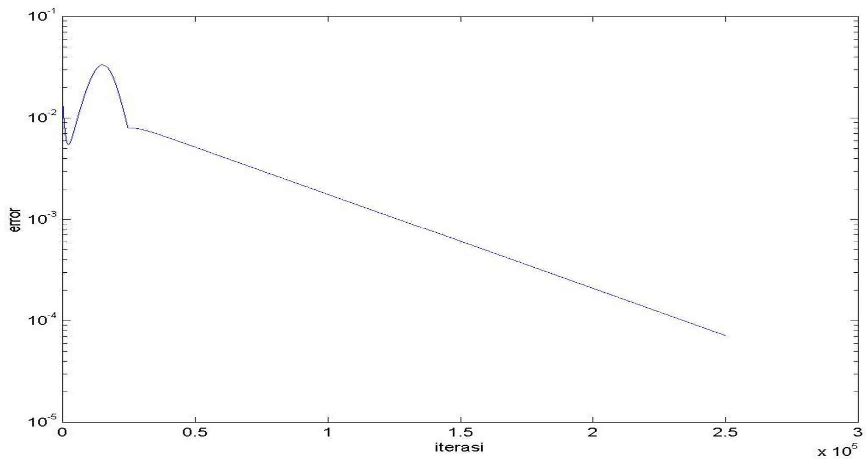

Figure 4: Graph Error of ABM 9 on the *Rabies* Virus Transmisson Model (withoutVaccination) at Iteration 250000

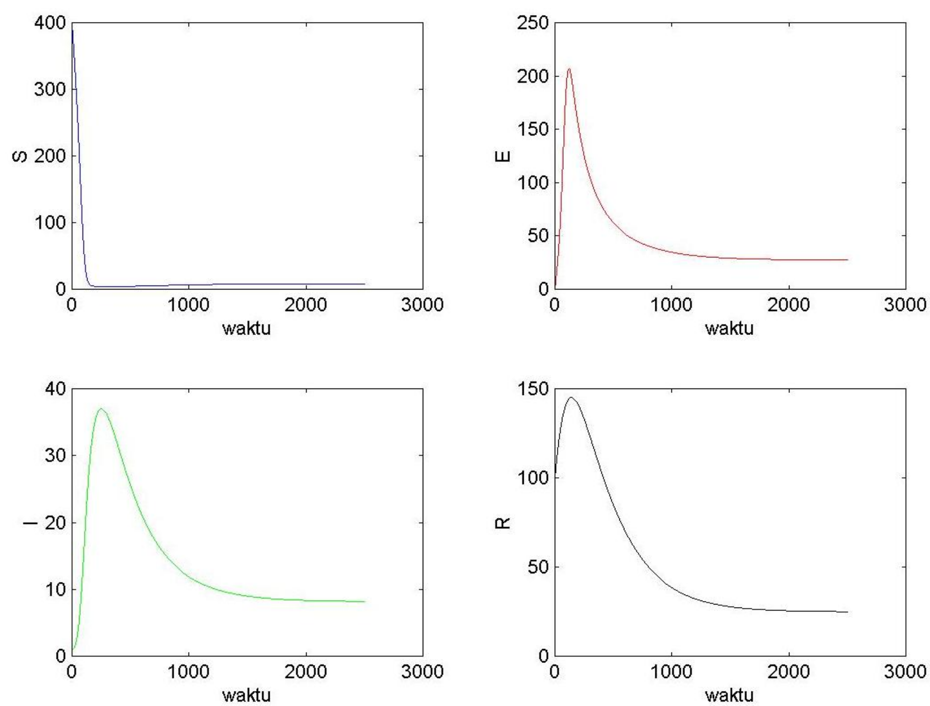

Figure 5: Graph Execution of ABM 12 (with vaccination) with Iteration 250000

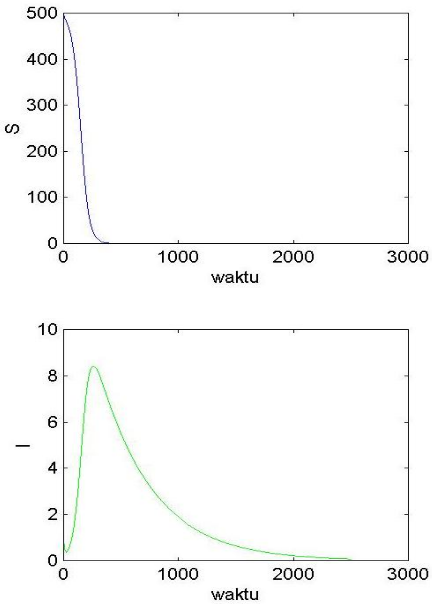

Figure 6: Graph Execution of ABM 9 (with vaccination) with Iteration 250000

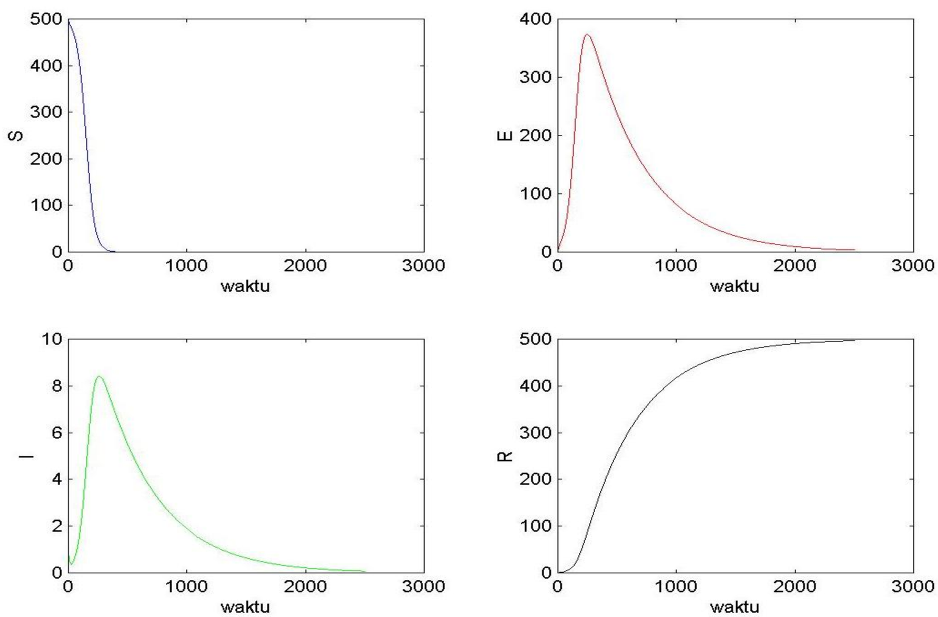

Figure 7: Graph Execution of ABM 12 (without vaccination) with Iteration 250000

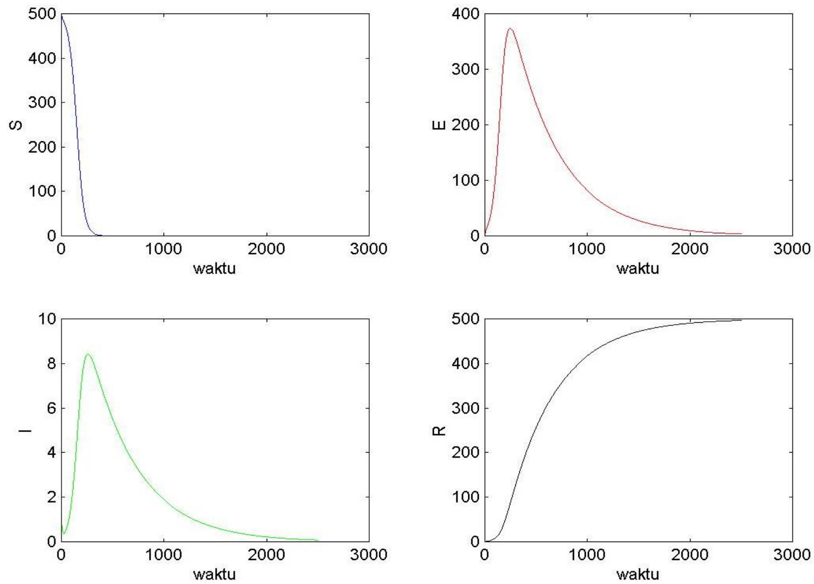

Figure 8: Graph Execution of ABM 9 (without vaccination) with Iteration 250000

Besides the graph also obtained data about error value of both methods at certain iteration, presented in the following table:

Tabel 3: Table of Error Execution at Same Iteration (with Vaccinationin Dogs)

Iteration	Error Value of ABM 9	Error Value of ABM 12
1000	0.02730837	0.02730875
5000	0.03693386	0.03697012
10000	0.026590041	0.026585188
50000	0.001636480	0.001636479
100000	0.000416860	0.000416820
250000	0.0000039228	0.0000039215

Tabel 4: Table of Error Execution at Same Iteration (withoutVaccination in Dogs

Iteration	Error Value of ABM 9	Error Value of ABM 12
1000	0.00709776	0.00709815
5000	0.00872863	0.00873226
10000	0.02233629	0.02234830
50000	0.00519084	0.00519089
100000	0.00177980	0.00177927
250000	0.00007158	0.000071495

The Results programming of Adams Bashforth-Moulton method of order 12 is the form of data error value and graphs Rabies virus infection based on mathematical model Rabies virus transmission dynamics in *SEIR* "*An Mathematical Model for Dog Rabies, Case Study: Bongo District, Ghana*" by Kwakku Mari Addo. Format programming carried on the parameters and initial values are relevant to the problem transmission of rabies virus in a similar report. Results of simulations presented in the different iterations in iteration 1000, 5000, 10000, 50000, 100000, and 250000 (In this article just show the graph execution at 250000 iteration),and divided into two different types of treatment namely the transmissionof rabies virus by vaccination and without vaccination. In the event ofvaccination, the number of dogs exposed to the virus (*susceptible*) continued tofall almost to zero at iteration 50000 and then increased to the level of a verysmall increase to day-to-2500 or 250000 iterations.

Meanwhile, the number of dogs latent (*exposed*) increasing from day-0 and thengo up on day-100 and and then the go down back to the constant on Day-2500.Total population of dogs infected (*infected*) increased from initial observationsup to 250 days and then went down to the constant is at a point 10 on the day2500. For the number of dogs treated (recovered) increased not so great untilday 50 and then continued to fall until constant 25 points on day 2500. On thetransmission of rabies virus in dogs without vaccination, the number of dogsthat are prone (*susceptible*) decreased very small until the day in 2500. As forthe number of dogs latent (*exposed*) continued to rise until day 200 and thencontinue down to day-to-2500, as well as the number of dogs infected with avirus (*infected*) which decreased at the beginning of the observation and thenrose after the day to-200 and eventually fall back to the point 0 on day 2500. Asfor the number of dogs treated (*recovered*) continues to increase to day-to-2500.

Based on the execution results of graph Adams Bashforth-Moulton method oforder 12 and Adams Bashforth-Moulton methods of order 9, it can be said that13the two graphs are similar, but when examined more deeply, both graphic differencesin error values achieved despite differences it's not much different. Ingeneral, based on the error value of both method on the similar iteration, it appearsthat the error rate (the value of error) produced by the Adams-BashforthMoulton order 12 smaller than the error value generated by Adams-BashforthMoulton order 9. Thus, it can be said that the

Adams-Bashforth Moulton order12 more effective than the methods of Adams-Bashforth Moulton order 9.

CONCLUSION AND SUGESTIONS

Based on the results of the discussion, can be conclude that:

1. the formula of Adams Bashforth-Moulton method order 12are:

   The formula of Adams Bashforth method as a predictor, is:

$$y_{n+12} = y_{n+11} + \frac{h}{958003200}(4527766399f_{n+11} - 19433810163f_{n+10} +61633227185f_{n+9} - 135579356757f_{n+8} + 214139355366f_{n+7} -247741639374f_{n+6} + 211103573298f_{n+5} - 131365867290f_{n+4} +58189107627f_{n+3} - 17410248271f_{n+2} + 3158642445f_{n+1} -262747265f_n)$$

The formulation of Adams Moulton method as a corrector is:

$$y_{n+12} = y_{n+11} + \frac{h}{958003200}(262747265f_{n+12} + 1374799219f_{n+11} -2092490673f_{n+10} + 3828828885f_{n+9} - 5519460582f_{n+8} +6043521486f_{n+7} - 4963166514f_{n+6} + 3007739418f_{n+5} -1305971115f_{n+4} + 384709327f_{n+3} - 68928781f_{n+2} +5675265f_{n+1}$$

2. Theoretically, the Linear Multistep Method of Adams Bashforth-Moultonorder 12is a convergent method.
3. The Linear Multistep Method of Adams Bashforth-Moulton order 12ismore effective than Adams Bashforth-Moulton order 9based on the errorvalue from the execution results of both method at the same iteration.

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