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SEMIPARAMETRIC MODELING OF CONSUMER PRICE INDEX

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Abstract. Many classical data, for example, exchange rate, stock price, and consumer price index (CPI) data cannot be analyzed under independent observation assumption. In addition, some time series data cannot be modeled well into a fully linear model, for instance, CPI, price of raw materials for some certain industries and price of some industrial products data in which monetary crisis of Indonesia in 1998 has caused a dramatic effect on the time series of CPI, price of raw materials and industrial products. A semiparametric model is a mixture model between parametric and nonparametric models. If we apply it to time series data, we will obtain a semiparametric time series model known as partly linear autoregressive model:

$$y_t = \beta \; y_{t-1} + g(y_{t-2},...,y_{t-p}) + \varepsilon_t \text{ for } t \geq p+1.$$

Here β is an unknown parameter to be estimated, $g(.)$ is an unknown function in R^{p-1}, ε_t are i.i.d. random errors with $E(\varepsilon_1) = 0$ and $E(\varepsilon_1^2) < \infty$, and ε_t are independent of y_s for all $s = 1,2,...,p$ and $t \geq p+1$. Based on the model above, we investigate a model for general consumer price index (GCPI) of Jember data recorded monthly from January 1998 to December 2002 by Statistic Center Bureau of Jember.

Keywords: Least Square Estimator, Kernel Estimator, Semiparametric Model, Consumer Price Index
MSC 2020 : 62F99

1. Introduction

In last twenty years, many authors have shown interest in semiparametric regression models, especially, partly linear regression models, $y_j = x_j'\beta + g(t_j) + \varepsilon_j$ for $j = 1,2,...$; e.g. Ansley and Wecker (1983), Heckman (1986), Rice (1986), Chen (1988), Chen and Shiau (1991), and Andrews (1991) investigated asymptotic behaviors by using spline smoothing techniques and Robinson (1988), Speackman (1988), Gao (1992), and Gao, et al (1994) also obtained some asymptotic results by using kernel or nearest neighbor smoothing under the case where the (x_j, t_j) are i.i.d. random variables or fixed design points, and Wahba (1990), Budiantara (1999) and Hardle (2000) studied partly linear model for independent observations case. However, many classical data, for example, exchange rate data, stock price data, and consumer price index data, cannot be analyzed under independent observation assumption. In addition, some time series data cannot be modeled as a fully linear model, for instance, consumer price index (CPI) data recorded monthly from January 1998 to December 2002 by Statistic Center Bureau of Jember in which monetary crisis of Indonesia in 1998 has caused a dramatic effect on the time series of CPI. Lestari (2001), Lestari (2003), Lestari (2004), and Lestari (2005) studied estimation, asymptotic normality and iterated logarithm distribution of partly linear autoregressive estimators; and applied partly linear autoregressive model to the consumer price index of Jember data. Beside that, latest development in the semiparametric regression has given a strong foundation to semiparametric time series analysis.

The important thing is how we analyze and apply semiparametric regression model to time series data. The resulting model is known as partly linear autoregressive model given by :

$$y_t = \beta\, y_{t-1} + g(y_{t-2},....,y_{t-p}) + \varepsilon_t ,\;\; t \geq p+1 \qquad (1)$$

where β is an unknown parameter to be estimated, $g(.)$ is an unknown function in R^{p-1}, ε_t are i.i.d. random errors with $E(\varepsilon_1) = 0$ and $E(\varepsilon_1^2) < \infty$, and ε_t are independent of y_s for all $s = 1,2,..., p$.

In this paper, we investigate a semiparametric time series model for the general consumer price index (GCPI) of Jember data which is known as partly linear autoregressive model.

2. Methodology

The model (1) contains unknown parameter β and unknown function $g(.)$ to be estimated. Least square method is used to obtain estimator $\hat{\beta}_T$ of β. The estimate of $g(.)$ can be obtained by using kernel estimator approach. In association with the GCPI data, of course, Firstly we identify the parametric and nonparametric components of data. Secondly, we determine an optimum bandwidth for kernel estimator approach with generalized cross validation (GCV) criterion and Gaussian kernel function. Finally, we estimate the model of GCPI by investigating and comparing their mean square errors (MSE), errors and plot for three models approach, i.e., linear autoregressive model, ARIMA model, and partly linear autoregressive model. For estimating and plotting, we use MINITAB, S-PLUS, and MATLAB programs.

3. Results

3. 1. Estimation of Function $g(.)$ and Parameter β

Assume that $\{y_t, t = p+1, p+2, ..., T\}$ satisfy the partly linear autoregressive model :

$$y_t = \beta y_{t-1} + g(y_{t-2}, ..., y_{t-p}) + \varepsilon_t \text{, t} \geq \text{p+1}. \qquad (2)$$

Then, we have :

$$\begin{aligned} g(y_{t-2}, ..., y_{t-p}) &= E((y_t - \beta y_{t-1}) | \; y_{t-2}, ..., y_{t-p}) \\ &= E(y_t | \; y_{t-2}, ..., y_{t-p}) - \beta E(y_{t-1} | \; y_{t-2}, ..., y_{t-p}) \\ &= g_1(y_{t-2}, ..., y_{t-p}) - \beta g_2(y_{t-2}, ..., y_{t-p}) \text{ (say)} \qquad (3) \end{aligned}$$

Hence, the natural estimates of $g_1(.)$, $g_2(.)$ and $g(.)$ of (3) can be obtained by using kernel estimator approach defined by :

$$\hat{g}_{1T}(x_t) = \sum_{s=p+1}^{T} W_{Ts}(x_t) y_s , \qquad (4)$$

$$\hat{g}_{2T}(x_t) = \sum_{s=p+1}^{T} W_{Ts}(x_t) y_{s-1} , \qquad (5)$$

and

$$\hat{g}_T(x_t) = \hat{g}_{1T}(x_t) - \beta \hat{g}_{2T}(x_t) \qquad (6)$$

where $x_t = (y_{t-2}, ..., y_{t-p})$ and $W_{ts}(.)$ are kernel weight function Nadaraya-Watson given by :

$$W_{Ts}(x) = K((x - x_s)/h) \Big/ \sum_{t=p+1}^{T} K((x - x_t)/h) \qquad (7)$$

where $K : R^{p-1} \to R$ is a function and $\{h = h_T ; T > p\}$ is a sequence of nonnegative real numbers.

Now, based on the model $y_t = \beta y_{t-1} + \hat{g}_T(y_{t-2}, ..., y_{t-p}) + \varepsilon_t$, $t = p+1, ..., T$, we have the least square estimator $\hat{\beta}_T$ of β :

$$\hat{\beta}_T = \frac{\sum_{t=p+1}^{T} (y_{t-1} - \hat{g}_{2T}(y_{t-2}, ..., y_{t-p}))(y_t - \hat{g}_{1T}(y_{t-2}, ..., y_{t-p}))}{\sum_{t=p+1}^{T} (y_{t-1} - \hat{g}_{2T}(y_{t-2}, ..., y_{t-p}))^2} \qquad (8)$$

3.2. Investigation of GCPI Model.

Based on the model (1), we investigate a model for general consumer price index (GCPI) data recorded monthly from January 1998 to December 2002 by Statistic Center Bureau of Jember. The GCPI data and their series plot are shown in Table 1 and Figure 1, respectively.

Table 1. General consumer price index from january 1998 to december 2001

Year	GCPI	Year	GCPI	Year	GCPI	Year	GCPI	Year	GCPI
1998	183.69	1999	187.8	2000	196.99	2001	198	2002	218.92
	185.86		188.01		197.12		199.87		217.1
	186.65		190.45		197.23		197.28		210.27
	192.55		193.1		197.29		199.67		218.07
	195.14		194.25		197.37		198.83		219.48
	194.77		192.35		197.4		199.54		219.96
	193.35		193.5		197.5		199.79		218.88
	192.75		195.2		197.73		199.92		216.89
	193.1		196.37		197.76		200.21		210.09
	189.92		195.88		197.84		218.09		216.25
	188.1		198.22		197.9		219.28		215.59
	186.69		196.41		197.95		220		219.06

Source : *Statistic Center Bureau of Jember*

3.2.1. Autoregressive Model Approach

In this approach, we consider a linear model as follows :

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + \varepsilon_t \,. \tag{9}$$

The linear regression analysis result on the data gives an estimated model :

$$\hat{y}_t = 8.81 + 0.923 y_{t-1} + 0.036 y_{t-2} \tag{10}$$

Validation result of $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ shows that $\hat{\beta}_1$ is significant, and $\hat{\beta}_0$ and $\hat{\beta}_2$ are not significant. Plot between y_t and $\hat{y}_t$ is shown in Figure 2.

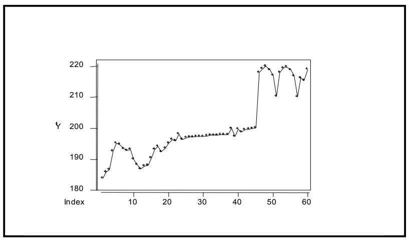


Figure 1. Series Plot of GCPI Data

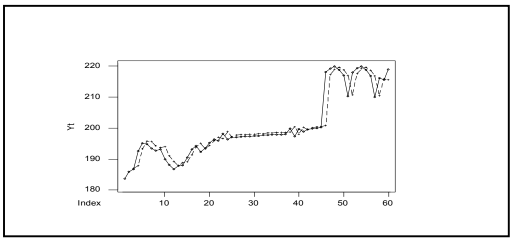


Figure 2. Plot Between y_t (—) and $\hat{y}_t = 8.81 + 0.923 y_{t-1} + 0.036 y_{t-2}$ (---)

In addition, plotting of residuals as shown model (10) shows that there are a right skew, outliers and indicates no normal distribution. Next, by getting out variables with no significant coefficients from the model, we obtain :

$$y_t = 1.00287 y_{t-1} + \varepsilon_t \tag{11}$$

Plotting of residuals model (11) shows that there are a right skew, outliers and indicates no normal distribution. Plot between y_t and $\hat{y}_t = 1.00287 y_{t-1}$ is shown in Figure 3. Residuals plots for models (10) and (11) are shown in Figure 4 and Figure 5, respectively.

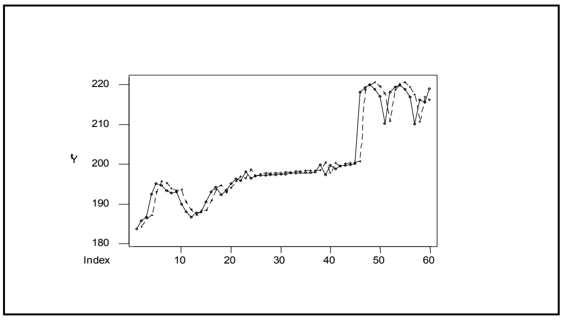
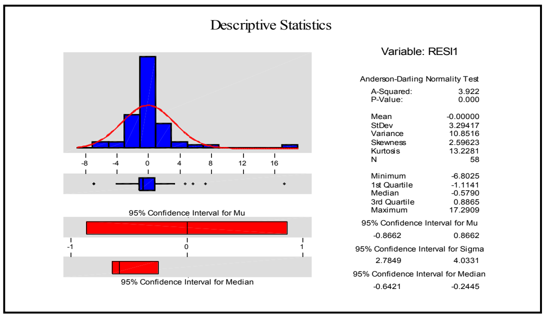


Figure 3. Plot between y_t and $\hat{y}_t = 1.00287 y_{t-1}$

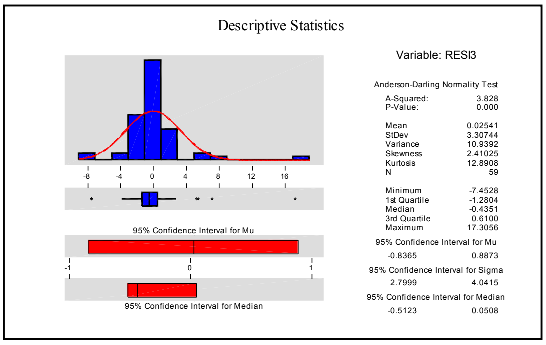


```
Variable: RESI1

Anderson-Darling Normality Test
  A-Squared:            3.922
  P-Value:              0.000

  Mean               -0.00000
  StDev               3.29417
  Variance            10.8516
  Skewness            2.59623
  Kurtosis            13.2281
  N                        58

  Minimum             -6.8025
  1st Quartile        -1.1141
  Median              -0.5790
  3rd Quartile         0.8865
  Maximum             17.2909
95% Confidence Interval for Mu
  -0.8662              0.8662
95% Confidence Interval for Sigma
   2.7849              4.0331
95% Confidence Interval for Median
  -0.6421             -0.2445
```

Figure 4. Residuals plot for Model (10)

3.2.2. ARIMA Model Approach

Figure 1 shows a series with trend. It means that the series has not been stationary yet. Next, by using difference $d = 1$, i.e., $w_t = y_t - y_{t-1}$, we have series plot of w_t as shown in Figure 6. Figure 6 shows that data has been stationary. Based on the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots in which they cut off after lag 5 and lag 6, we have the estimated model of **ARIMA(1,1,0)(1,0,0)^6**



```
Variable: RESI3

Anderson-Darling Normality Test
  A-Squared:            3.828
  P-Value:              0.000

  Mean                0.02541
  StDev               3.30744
  Variance            10.9392
  Skewness            2.41025
  Kurtosis            12.8908
  N                        59

  Minimum             -7.4528
  1st Quartile        -1.2804
  Median              -0.4351
  3rd Quartile         0.6100
  Maximum             17.3056
95% Confidence Interval for Mu
  -0.8365              0.8873
95% Confidence Interval for Sigma
   2.7999              4.0415
95% Confidence Interval for Median
  -0.5123              0.0508
```

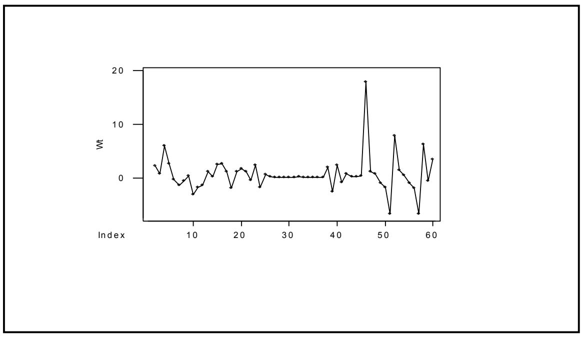


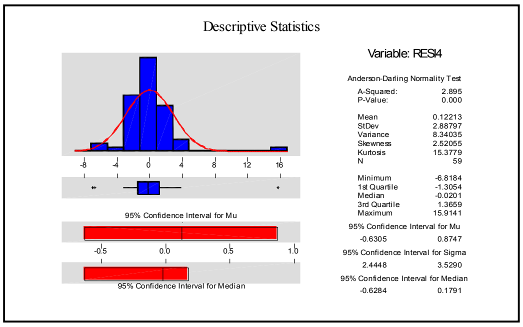
Figure 5. Residuals plot for model (11)

Figure 6. Series plot with difference $d = 1$

Although diagnostic checking result shows that residuals has been white noise but estimation of parameters result shows that SAR parameter is significant different from zero and AR parameter is not significant. Plot of residuals normality shows that residuals are not normal and have outliers as shown in Figure 7.

3.2.3. Semiparametric Model Approach

By plotting between y_t and y_{t-1}; and between y_t and y_{t-2} we knew that plotting result between y_t and y_{t-1} was relatively linear (Figure 8). On the other hand, plotting result between y_t and y_{t-2} was not linear and had no certain pattern (Figure 9). Therefore, y_{t-1} and y_{t-2} were as parametric and nonparametric components, respectively.



```
Variable: RESI4

Anderson-Darling Normality Test
  A-Squared:          2.895
  P-Value:            0.000

  Mean                0.12213
  StDev               2.88797
  Variance            8.34035
  Skewness            2.52055
  Kurtosis            15.3779
  N                   59

  Minimum             -6.8184
  1st Quartile        -1.3054
  Median              -0.0201
  3rd Quartile         1.3659
  Maximum             15.9141
95% Confidence Interval for Mu
  -0.6305             0.8747
95% Confidence Interval for Sigma
  2.4448              3.5290
95% Confidence Interval for Median
  -0.6284             0.1791
```

Figure 7. Residuals plot for ARIMA Model

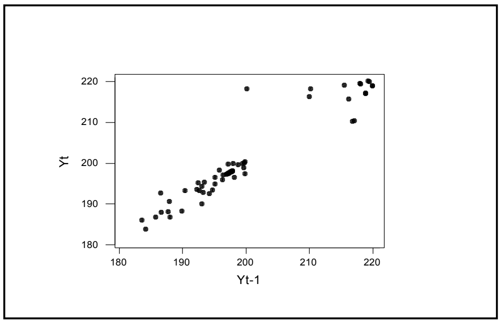


Figure 8. Plot between y_t and y_{t-1}

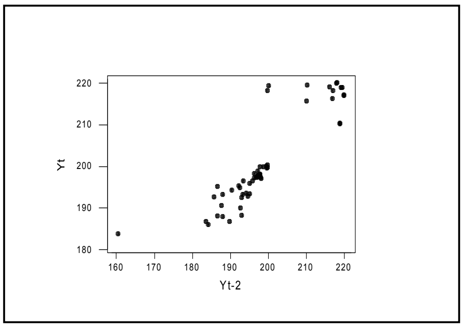


Figure 9. Plot between y_t and y_{t-2}

It is also supported by linear regression plot results between y_t and y_{t-1} with $R^2 = 90.6$ %, and between y_t and y_{t-2} with $R^2 = 78.6$ % as shown in Figure 10 and Figure 11, respectively.

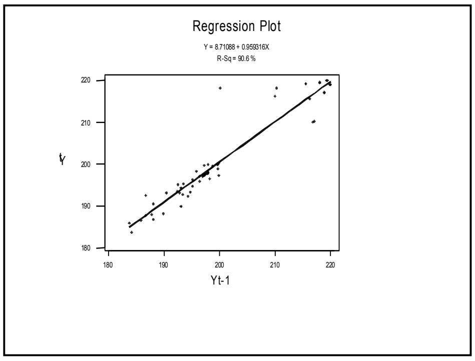


Figure 10. Regression plot between y_t and y_{t-1}

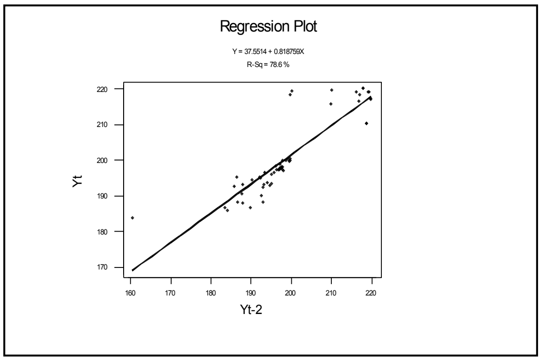


Figure 11. Regression plot between y_t and y_{t-2}

After determining both parametric and nonparametric components, we find the optimum bandwidth for Gaussian kernel estimator approach with the generalized cross validation (GCV) criterion. So that, by using programs Matlab and Minitab, we obtained some bandwidth and GCV values as shown in Table 2 and Figure 12, respectively. Table 2 and

Figure 12 show that the optimum bandwidth is 0.017, because GCV minimum, i.e. 1.8447, is obtained at bandwidth (h) = 0.017.

Next, based on that optimum bandwidth we estimated $\hat{g}_T(y_{t-2})$, $\hat{\beta}_T$, $\hat{y}_t$ and ε_t values by using program Matlab. We got $\hat{\beta}_T = 0.9192$; and $\hat{y}_t$, $\hat{g}_T(y_{t-2})$ and ε_t as shown in Table 3. It shows that errors are relatively small or leads to zero for every t. So, we obtained the mean square error (MSE), i.e., 0.007083. The relationship of $\hat{y}_t$, y_{t-1} and y_{t-2} formed a surface as shown in Figure 13.

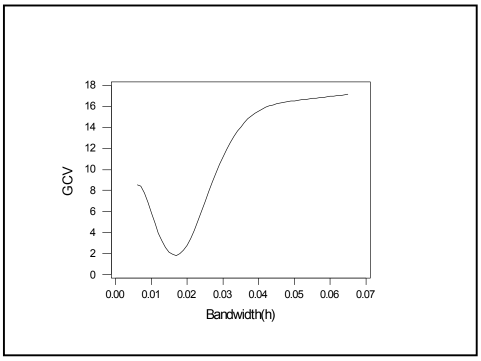


Figure 12. GCV versus bandwidth plot

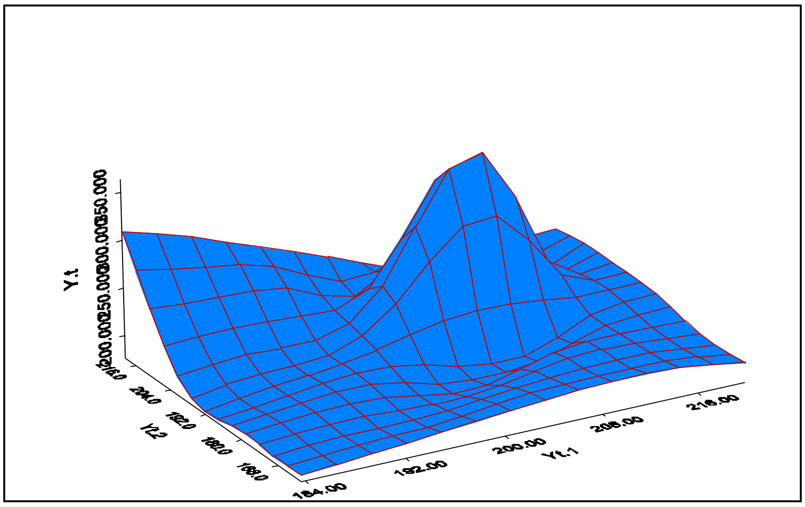


Figure 13. Surface performed by $\hat{y}_t$, y_{t-1} and y_{t-2}

Next, based on that optimum bandwidth we estimated $\hat{g}_T(y_{t-2})$, $\hat{\beta}_T$, $\hat{y}_t$ and ε_t values by using program Matlab. We got $\hat{\beta}_T = 0.9192$; and $\hat{y}_t$, $\hat{g}_T(y_{t-2})$ and ε_t as shown in Table 3. It shows that errors are relatively small or leads to zero for every t. So, we obtained the mean square error (MSE), i.e., 0.007083. The relationship of $\hat{y}_t$, y_{t-1} and y_{t-2} formed a surface as shown in Figure 13.

Table 2. Some GCV values for some bandwidths

Bandwidth	GCV	Bandwidth	GCV
0.01	5.83	0.017	1.8447
0.011	4.8474	0.018	1.9758
0.012	3.9612	0.019	2.3006
0.013	3.2064	0.02	2.8035
0.014	2.6038	0.021	3.4589
0.015	2.1676	0.022	4.2347
0.016	1.9107	0.023	5.0963

Table 3. Estimated values of $\hat{y}_t$, $\hat{g}_T(y_{t-2})$ and ε_t

No	$\hat{y}_t$	$\hat{g}_T(y_{t-2})$	ε_t	No	$\hat{y}_t$	$\hat{g}_T(y_{t-2})$	ε_t
1	183.685	14.3408	0	31	197.519	16.0686	-0.024
2	185.855	17.0072	0	32	197.701	16.1586	0.024
3	186.645	15.8024	0	33	197.755	16.0012	0
4	192.545	20.9762	0	34	197.832	16.0513	0.0024
5	194.972	17.9796	0.1632	35	197.897	16.0425	-0.0024
6	194.765	15.392	0	36	197.945	16.035	0
7	193.347	14.3142	-0.0021	37	198.018	16.0627	-0.0238
8	192.745	15.0174	0	38	199.786	17.784	0.079
9	193.095	15.919	0	39	197.331	13.6103	-0.0563
10	189.915	12.4172	0	40	199.87	18.5302	-0.2053
11	188.23	13.6553	-0.1349	41	199.14	15.6035	-0.3156
12	186.685	13.7834	0	42	199.535	16.7701	0
13	187.795	16.1895	0	43	199.785	16.3674	0
14	188.168	15.5424	-0.1632	44	199.915	16.2676	0
15	190.445	17.6261	0	45	200.205	16.4381	0
16	193.095	18.0332	0	46	217.879	33.8462	0.2053
17	194.245	16.7472	0	47	219.274	18.8057	0
18	192.21	13.6553	0.1349	48	219.919	18.3572	0.0747
19	193.495	16.6867	0	49	218.914	16.69	0
20	195.195	17.3295	0	50	217.084	15.8526	0.0102
21	196.365	16.9369	0	51	210.265	10.7066	-0.0008
22	195.873	15.3693	0.0021	52	218.064	24.7841	0
23	198.089	18.0365	0.1252	53	219.474	19.0241	0
24	196.405	14.2008	0	54	220.029	18.2827	-0.0747
25	196.985	16.4446	0	55	218.874	16.6868	0
26	197.24	16.1667	-0.1252	56	216.894	15.6998	-0.0102
27	197.225	16.0319	0	57	210.083	10.7181	0.0008
28	197.285	15.9908	0	58	216.244	23.1296	0
29	197.355	16.0061	0.0096	59	215.584	16.8071	0
30	197.082	15.6599	0.3122	60	219.054	20.8838	0

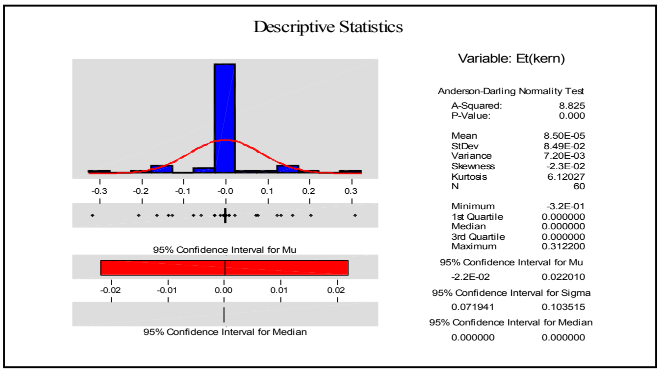
3.3. Comparing of Three Models Approach

Based on the discussion above, we can compare mean square errors (MSE), Errors (ε_t) and plots for these three models approach, i.e., autoregressive, ARIMA and semiparametric (partly linear autoregressive) models as follows:

(i). MSE of autoregressive, ARIMA, and semiparametric models are 11, 8.502 and 0.007083, respectively. We can see that MSE of semiparametric model approach is smaller than both autoregressive and ARIMA models approach.

(ii). Errors (ε_t) of both autoregressive and ARIMA models approach are not normal, but errors (ε_t) of semiparametric model approach is normal as shown in Figure 14. However, these three models approach satisfy white noise condition.

(iii).Plot between estimated values ($\hat{y}_t$) and observation values (y_t) for autoregressive, ARIMA and semiparametric models approach are given by Figure 15, Figure 16, and Figure 17, respectively. We can see that plot result of semiparametric model approach gives estimated values ($\hat{y}_t$) exactly close to observation values (y_t). It is different from both autoregressive and ARIMA models approach.



Descriptive Statistics

```
Variable: Et(kern)

Anderson-Darling Normality Test
  A-Squared:          8.825
  P-Value:            0.000

  Mean             8.50E-05
  StDev            8.49E-02
  Variance         7.20E-03
  Skewness         -2.3E-02
  Kurtosis          6.12027
  N                      60

  Minimum          -3.2E-01
  1st Quartile     0.000000
  Median           0.000000
  3rd Quartile     0.000000
  Maximum          0.312200

95% Confidence Interval for Mu
  -2.2E-02         0.022010
95% Confidence Interval for Sigma
  0.071941         0.103515
95% Confidence Interval for Median
  0.000000         0.000000
```

-0.3
-0.2
-0.1
-0.0
0.1
0.2
0.3
95% Confidence Interval for Mu
-0.02
-0.01
0.00
0.01
0.02
95% Confidence Interval for Median

Figure 14. Residuals plot for semiparametric Model.

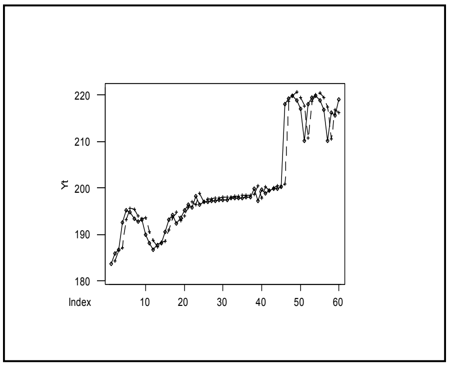


Figure 15. Series plot between y_t and $\hat{y}_t$ for autoregressive model

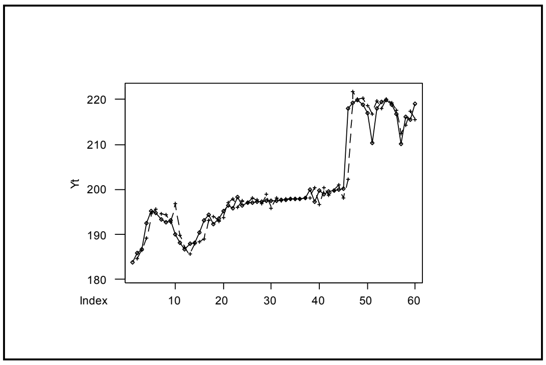


Figure 16. Series plot between y_t and $\hat{y}_t$ for ARIMA model.

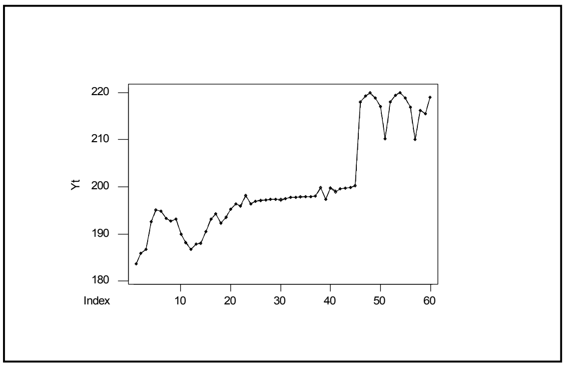


Figure 17. Series plot between y_t and $\hat{y}_t$ for semiparametric model.

4. Conclusion

Based on the discussion above, we known that using of semiparametric model approach for estimating a model of general consumer price index of Jember is better than both autoregressive and ARIMA models approach. We obtained an estimated model of general consumer price index of Jember as a semiparametric time series model which is called as partly linear autoregressive model as follows :

$$\hat{y}_t = 0.9192 y_{t-1} + \hat{g}_T(y_{t-2}) \qquad (12)$$

Where $\hat{g}_T(y_{t-2}) = \hat{g}_{1T}(y_{t-2}) - 0.9192\hat{g}_{2T}(y_{t-2})$,

$$\hat{g}_{1T}(y_{t-2}) = \sum_{s=3}^{T} W_{Ts}(y_{t-2}) y_s,$$

$$\hat{g}_{2T}(y_{t-2}) = \sum_{s=3}^{T} W_{Ts}(y_{t-2}) y_{s-1},$$

$$W_{Ts}(y_{t-2}) = K\left((y_{t-2} - y_s)h^{-1}\right) \Big/ \sum_{j=3}^{T} K\left((y_{t-2} - y_j)h^{-1}\right) \text{ and}$$

$$K(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}, \quad -\infty < y < \infty.$$

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