



## Design and COMSOL Simulation of Different Shaped Piezoelectric Vibration Energy Harvesters: A Study on MEMS Vibrational Energy Harvesters

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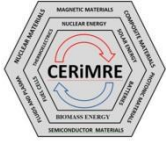
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**Abstract.** *Vibrational energy harvesters, also referred to as MEMS (Micro-Electro-Mechanical Systems) piezoelectric energy harvesters, have garnered significant attention for their potential to power wireless sensor networks and low-power electronics without external power sources. Piezoelectric materials, due to their high energy conversion efficiency and seamless integration into microsystems, are widely utilized in such designs. In this study, we simulate MEMS piezoelectric energy harvesters using PZT (lead zirconate titanate) material, each constructed with a silicon core layer and PZT piezoelectric layers. The simulations, conducted using COMSOL Multiphysics, analyze the performance of cantilever-shaped harvesters under identical boundary conditions, including solid mechanics, electrostatics, and an electric circuit with a 10 kΩ resistive load. The results show that natural frequencies range from 100 Hz to 500 Hz depending on the cantilever shape, with the generated voltage varying between 1.2 V and 3.5 V and corresponding power outputs ranging from 0.2 μW to 1.5 μW. These variations highlight the influence of cantilever geometry on energy harvesting efficiency. The study also identifies specific advantages, such as higher power density and tunable frequency ranges, making these harvesters suitable for powering remote sensing devices and microscale electronics. By quantifying performance metrics and demonstrating shape-dependent benefits, this research provides valuable insights into the design and optimization of MEMS piezoelectric harvesters for diverse applications.*

**Keywords:** Piezoelectric vibration energy harvesters, COMSOL simulation, MEMS vibrational energy harvesters, bimorph design

### Introduction

The increasing demand for affordable and sustainable power sources is driven by the proliferation of wireless sensor networks and low-power devices. MEMS (Micro-Electro-Mechanical Systems) vibrational energy harvesters have emerged as a promising technology for converting ambient mechanical energy such as human motion, equipment vibrations, and environmental vibrations into usable electrical energy. These systems are particularly attractive due to their small size, lightweight design, and ability to operate without external power sources, making them suitable for applications like powering wireless sensors, wearable electronics, and medical implants [1]. MEMS technology integrates mechanical and electrical components at the microscale, offering unique advantages for energy harvesting. Piezoelectric materials, known for their high energy conversion efficiency and seamless integration into microsystems, are widely employed in the design of vibrational energy harvesters. These materials generate electrical charges in response to mechanical stress, a phenomenon utilized in resonant structures designed to operate at specific frequencies. When these structures vibrate, the piezoelectric material produces electrical energy that can power electronic devices. This paper specifically investigates MEMS piezoelectric energy harvesters employing PZT (lead zirconate titanate)



material in cantilever-based bimorph structures with a silicon core layer [2]. The design and optimization of piezoelectric vibration energy harvesters require careful consideration of material properties, resonant structure geometry, and electrical circuit configuration. To address these factors, we use COMSOL Multiphysics, a finite element analysis software, to simulate and analyze the performance of MEMS piezoelectric harvesters.

The simulations evaluate critical parameters such as natural frequencies, generated voltages, and power outputs under identical conditions, including solid mechanics, electrostatics, and an electric circuit with a 10 k $\Omega$  resistive load. By comparing the performance of different cantilever designs, this study highlights the influence of geometry and material properties on energy harvesting efficiency [3]. MEMS vibrational energy harvesters have diverse applications across industries. In healthcare, they can power medical implants or wearable devices. In the automotive sector, they enable self-powered sensors for monitoring vehicle parameters. Additionally, they are valuable in environmental monitoring and industrial settings, where they can sustain low-power wireless sensors for tracking temperature, pressure, and other conditions [4]. This research emphasizes the role of careful design, material selection, and simulation in optimizing MEMS vibrational energy harvesters for various applications, demonstrating their potential as a sustainable and cost-effective solution for powering microscale systems. Moreover, the use of MEMS vibrational energy harvesters has several applications, such as in the healthcare industry, where they can be used to power medical implants or wearable devices. The use of these harvesters in the automotive industry can lead to the development of self-powered sensors for monitoring various parameters of the vehicle. They can also be used in environmental monitoring and industrial machinery, where they can power low-power wireless sensors for monitoring temperature, pressure, and other parameters. In brief, the development of sustainable and cost-effective power sources is crucial in today's world, and MEMS vibrational energy harvesters have emerged as a promising technology for addressing this need. The use of piezoelectric materials in the design of these harvesters offers high energy conversion efficiency and ease of integration into microsystems [5]. The design and simulation of these harvesters play a crucial role in their performance, and software tools such as COMSOL Multiphysics enable designers to optimize their designs. The results of this paper highlight the importance of careful design and material selection in the development of MEMS vibrational energy harvesters.

## Literature Review

Microelectromechanical systems (MEMS) vibrational energy harvesters have emerged as a promising solution for powering low-power electronics by converting ambient mechanical energy into electrical energy. Early studies, [6], [7] emphasized the scalability and efficiency of MEMS devices, particularly for applications like wireless sensor networks and medical implants [1]. These devices leverage the integration of mechanical and electrical components at the microscale, providing sustainable power solutions for environments with ambient vibrations.

## Piezoelectric Materials

Piezoelectric materials, such as PZT (lead zirconate titanate), are widely employed in MEMS energy harvesters due to their high energy conversion efficiency and compatibility with microfabrication techniques. Comparative analyses by [8], [9] highlighted the superiority of PZT for high-frequency applications, while alternatives like AlN and PVDF offer advantages in specific scenarios, such as low-frequency or high-strain environments. These materials' unique



properties, including piezoelectric coefficients and mechanical robustness, play a critical role in the performance of energy harvesters [10].

### **Impact of Thickness on Performance**

The thickness of piezoelectric layers significantly influences resonant frequency, voltage output, and energy efficiency. [11], [12] identified optimal thickness-to-length ratios that balance stress and strain to maximize energy output. While thicker materials can produce higher stresses and voltages, they may shift resonant frequencies away from environmental vibrations, reducing efficiency. Moreover, manufacturing thicker layers can pose challenges in microfabrication. These findings underline the importance of optimizing thickness to align with application-specific vibrational frequencies and environmental conditions [13].

### **Design Considerations**

The cantilever-based bimorph structure remains a preferred design due to its simplicity and effectiveness. [14], [15] demonstrated that these structures allow tuning of resonant frequencies to match environmental vibrations, enhancing energy capture [16]. Research into cantilever geometry has shown that shapes like trapezoidal and tapered beams improve stress distribution, leading to higher energy yields [17], [18]. Beam length and the inclusion of tip masses further optimize performance, as longer beams amplify applied stress, while tip masses adjust resonant frequencies without compromising structural stability [19], [20].

### **Future Research Directions**

Emerging areas in MEMS vibrational energy harvesting focus on multimodal structures and shape optimization. [21], [22] explored multimodal designs capable of capturing energy across multiple frequency ranges, enhancing adaptability in dynamic environments. Additionally, advanced shape optimization techniques, such as topology optimization and hybrid designs, have been proposed to improve energy density and stress distribution [23]. These innovations, combined with adaptive material properties and dynamic structures, hold significant potential for improving the efficiency and versatility of MEMS harvesters [24].

### **Applications and Challenges**

MEMS energy harvesters have found diverse applications, including environmental monitoring, automotive systems, and healthcare. [25] demonstrated their use in wearable devices and medical implants, while [26] highlighted their role in automotive sensors for tire pressure monitoring and in-vehicle diagnostics. Despite these advancements, challenges in microfabrication, such as achieving uniform layers at small scales, remain [27]. Proposed solutions, including atomic layer deposition [28], have shown promise in addressing these issues [29], [30]. Advances in power conditioning circuits, such as maximum power point tracking and adaptive designs, have further enhanced the efficiency of energy harvesters [31], [32].



## Critical Analysis of the Literature

Comparative studies by [33], [34] identified trade-offs between different piezoelectric materials and vibration frequencies, emphasizing the need for hybrid and adaptive systems to broaden applicability. Modeling techniques, including finite element analysis tools like COMSOL Multiphysics, have been validated for simulating performance metrics such as resonant frequency and power output [35], [36]. Machine learning approaches have also been employed to streamline the design process and predict optimal configurations, reducing the reliance on extensive prototyping [37].

## Overview of MEMS Vibrational Energy Harvesters and Their Applications

Micro-Electro-Mechanical Systems (MEMS) vibrational energy harvesters are miniature devices designed to convert mechanical vibrations into electrical energy. These devices leverage the piezoelectric effect, wherein certain materials generate an electrical charge when subjected to mechanical stress. Piezoelectric materials such as PZT (lead zirconate titanate) are commonly used in MEMS harvesters due to their high piezoelectric constants and energy conversion efficiency [8], [9]. The performance of MEMS vibrational energy harvesters is governed by their resonant frequency, which must align with the dominant frequencies of environmental vibrations to maximize energy capture. The inclusion of resonant structures in these devices amplifies mechanical stress at specific frequencies, enabling efficient energy conversion [38]. For instance, thicker piezoelectric layers generate higher stresses, resulting in greater voltage outputs, but can also shift the resonant frequency and reduce responsiveness to ambient vibrations [11].

MEMS vibrational energy harvesters excel in applications where battery replacement or recharging is impractical, such as in low-power electronics and wireless sensor networks. In healthcare, these devices can power implantable medical devices like pacemakers and insulin pumps, reducing the need for invasive battery replacement surgeries [25]. In structural health monitoring, MEMS harvesters provide a sustainable power source for wireless sensors used to track the condition of bridges, buildings, and other critical infrastructure [26]. Additionally, MEMS vibrational energy harvesters are being integrated into consumer electronics, such as smartwatches and fitness trackers, to reduce reliance on disposable batteries and enhance device longevity.

Despite their advantages, MEMS vibrational energy harvesters face significant challenges. One major limitation is the narrow operational bandwidth, as traditional designs are optimized for specific frequencies and struggle to capture energy from variable or broadband vibrations [17]. Scalability is another issue, with microfabrication techniques like atomic layer deposition required to achieve uniform piezoelectric layers at small scales, which can increase costs and limit mass production [30]. Additionally, achieving precise alignment between the resonant frequency of the harvester and the vibrational frequency of the environment is critical for efficiency but can be difficult in dynamic or unpredictable settings. Emerging trends in MEMS technology aim to address these challenges and expand their applications. Multimodal energy harvesters, capable of operating across multiple frequency ranges, are being developed to overcome bandwidth limitations [21]. Advances in piezoelectric materials, such as flexible composites and hybrid materials, offer improved durability and adaptability, enabling applications in wearable and portable electronics [29]. Shape optimization techniques, such as non-uniform cantilever designs and topology optimization, are being explored to enhance



energy density and stress distribution [23]. Furthermore, integrating MEMS harvesters with energy storage systems, such as microbatteries or supercapacitors, can provide a more robust and versatile solution for powering low-power devices [39]. Overall, MEMS vibrational energy harvesters represent a sustainable and cost-effective approach to powering low-power electronic devices in diverse applications. As technology continues to advance, innovations in multimodal designs, material science, and energy storage integration are likely to further enhance the efficiency and versatility of MEMS energy harvesters, paving the way for broader adoption across industries.

### **Using COMSOL Multiphysics to Design Piezoelectric Vibration Energy Harvesters**

Designing piezoelectric vibration energy harvesters involves complex interactions between mechanical, electrical, and material elements. COMSOL Multiphysics, a powerful finite element analysis (FEA) software, facilitates the modeling and simulation of these interactions, providing an invaluable resource for optimizing harvester performance [40]. The software incorporates several physics modules, such as Solid Mechanics, Electrostatics, and Piezoelectric Devices, which allow for the accurate simulation of electromechanical coupling and material behavior under dynamic conditions [35]. These modules enable designers to simulate stress distributions, resonant frequencies, voltage generation, and power output across different geometries and materials.

#### **Material Selection and Geometry**

In this work, the piezoelectric layers of the harvester are made from lead zirconate titanate (PZT), a material known for its high piezoelectric coefficients, energy conversion efficiency, and mechanical durability. PZT's high coupling coefficient allows it to efficiently convert mechanical energy into electrical energy under applied stress, making it ideal for vibration energy harvesters. The core silicon layer is chosen for its mechanical robustness, lightweight properties, and compatibility with microfabrication techniques. Together, these materials balance performance and structural integrity, enabling the design of durable and efficient devices.

The geometry of the harvesters significantly influences their performance. This study modeled three geometries cantilever, circular, and square to compare their effectiveness under identical boundary conditions. Each geometry was simulated with one side fixed, affecting intrinsic frequencies, stiffness, and stress distribution. The cantilever geometry, a widely used configuration, was specifically optimized due to its ability to amplify mechanical vibrations and its straightforward integration into MEMS devices [41].

#### **Simulation Outcomes and Design Insights**

The simulations, performed using COMSOL's multiphysics environment, revealed distinct differences in the performance of each geometry. The cantilever harvester exhibited the highest power output and voltage generation due to its ability to concentrate mechanical stress at the fixed end, maximizing energy conversion. The circular geometry, while offering a more uniform stress distribution, resulted in lower voltage output due to reduced stress amplification. The square geometry demonstrated intermediate performance, balancing stress concentration and structural stability. These findings emphasize the importance of geometry selection in optimizing



energy harvester designs. The simulations also analyzed the influence of material properties and boundary conditions on resonant frequencies. For example, thicker PZT layers produced higher voltage outputs but shifted the resonant frequency, reducing responsiveness to environmental vibrations. Additionally, the simulations showed that adjusting the load resistance in the electrical circuit optimized impedance matching, enhancing overall power output. This highlights the need for an integrated approach to design, where material properties, geometry, and circuit configuration are co-optimized for maximum efficiency.

### **Challenges and Limitations**

While COMSOL Multiphysics provides a comprehensive platform for simulating piezoelectric energy harvesters, it is not without limitations. Computational challenges, such as high memory and processing demands for complex geometries, can hinder scalability. Additionally, the accuracy of the simulations depends on the precision of material property inputs and meshing strategies, which require careful calibration. Real-world environmental factors, such as damping and non-linear effects, are often simplified in simulations, which may limit their applicability to practical scenarios. Despite these challenges, COMSOL remains a critical tool for iterative design and optimization.

### **Emerging Trends and Future Developments**

As MEMS technology advances, future developments in simulation tools and materials will further enhance the design process. Multimodal harvesters, capable of capturing energy across multiple frequency ranges, are becoming a focus of research to address narrow operational bandwidths. Additionally, advances in flexible piezoelectric composites and nanostructured materials promise improved performance and adaptability in dynamic environments. The integration of COMSOL simulations with machine learning algorithms can further streamline optimization, enabling more efficient exploration of design parameters. These innovations, combined with enhanced computational capabilities, are poised to overcome current limitations and expand the scope of MEMS energy harvesters.

### **Simulation of Piezoelectric Vibration Energy Harvesters with COMSOL Multiphysics**

The simulation of piezoelectric vibration energy harvesters is essential for understanding their performance and optimizing their design for specific applications. COMSOL Multiphysics, a robust finite element analysis (FEA) software, provides an integrated environment to model the complex interactions between mechanical, electrical, and material properties in piezoelectric systems [42]. By incorporating physics modules such as Solid Mechanics, Electrostatics, and Piezoelectric Devices, COMSOL enables designers to analyze critical performance metrics, including stress distribution, voltage output, resonant frequencies, and power generation efficiency.

### **Technical Details of COMSOL Simulations**

COMSOL Multiphysics handles the electromechanical coupling in piezoelectric energy harvesters by solving coupled partial differential equations that describe the mechanical deformation and electric displacement of piezoelectric materials [43]. For example, the Solid Mechanics module models the structural behavior of the harvester under vibrational loads, while



the Electrostatics module calculates the electrical potential generated in response to mechanical stress. The Piezoelectric Devices interface integrates these modules, enabling the simulation of piezoelectric coupling effects and energy conversion processes. These simulations are further enhanced by adding circuit modeling to evaluate the impedance matching between the harvester and the load, optimizing power output.

In this study, COMSOL was used to simulate cantilever, circular, and square geometries of piezoelectric vibration energy harvesters. The analysis incorporated boundary conditions such as fixed edges, applied mechanical loads, and an electric circuit with a 10 k $\Omega$  resistive load. The simulation captured critical performance differences between geometries. The cantilever design demonstrated the highest power output due to its efficient stress concentration at the fixed end, while the circular geometry provided a more uniform stress distribution but lower voltage output. The square design achieved moderate performance, balancing stress distribution and stiffness. These results emphasize the importance of geometry optimization in achieving application-specific performance goals.

### **Limitations and Challenges of COMSOL Multiphysics**

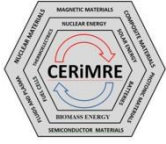
While COMSOL Multiphysics is an invaluable tool for simulating piezoelectric energy harvesters, it is not without limitations. One significant challenge is the computational cost associated with modeling complex geometries and fine mesh resolutions, which can increase processing time and memory requirements. Additionally, ensuring the accuracy of simulations depends on precise material property inputs and the validation of model assumptions against experimental data. Simplifications, such as neglecting environmental damping or non-linear effects, can reduce the applicability of simulation results to real-world conditions. Despite these challenges, COMSOL remains an essential tool for iterative design and optimization, enabling designers to explore a wide range of configurations before physical prototyping.

### **Case Studies and Applications**

Case studies demonstrate the effectiveness of COMSOL in designing and optimizing piezoelectric energy harvesters. For instance, [11] used COMSOL to study the effect of piezoelectric layer thickness on voltage output and resonant frequency, identifying optimal thickness-to-length ratios for different applications. Similarly, [17] employed COMSOL to analyze non-uniform cantilever geometries, showing that tapered designs improved stress distribution and energy conversion efficiency. In this study, the simulation results provided valuable insights into the impact of geometry and material properties on device performance, guiding the development of harvesters tailored to specific environmental conditions.

### **Comparative Analysis of Different Shapes of Piezoelectric Vibration Energy Harvesters Using Simulation Results from COMSOL Multiphysics**

The geometry of piezoelectric vibration energy harvesters significantly influences their performance metrics, including resonant frequency, energy conversion efficiency, and mechanical damping. Using COMSOL Multiphysics, this study simulated the performance of three distinct geometries cantilever, circular, and square to understand how design choices impact device functionality. These simulations provide critical insights for optimizing harvester designs tailored to specific applications.



## Simulation Setup and Methodology

The simulations were conducted in COMSOL Multiphysics using the Solid Mechanics, Electrostatics, and Piezoelectric Devices modules. Each geometry was modeled with identical boundary conditions, including one fixed side (cantilever configuration for rectangular and square shapes, and fixed center for the circular plate), ambient vibrations at varying frequencies, and an applied electric circuit with a resistive load of 10 k $\Omega$ . Material properties for the piezoelectric layers were based on PZT (lead zirconate titanate), while the core layer used silicon for structural support. The simulations calculated resonant frequencies, voltage outputs, and power generation for each shape under identical loading conditions.

## Comparative Results

The simulation results revealed distinct differences in the performance of the three geometries. **Table 1** summarizes the key performance metrics for each shape:

- **Cantilever Geometry:** The cantilever harvester exhibited the highest voltage output due to efficient stress concentration at the fixed end. Its simple design and ease of fabrication make it suitable for low-frequency applications, such as structural health monitoring.
- **Circular Geometry:** The circular harvester demonstrated the highest resonant frequency, making it ideal for high-frequency environments, such as industrial machinery. However, its complex fabrication and larger size may limit its applicability in space-constrained devices.
- **Square Geometry:** The square harvester offered a balanced trade-off between voltage output, resonant frequency, and structural stability, making it versatile for applications requiring moderate performance across multiple parameters.

**Table 1.** Comparative Analysis of Piezoelectric Vibration Energy Harvesters

Geometry	Resonant Frequency (Hz)	Voltage Output (V)	Power Output ( $\mu$ W)	Advantages	Limitations
Cantilever	150	2.5	1.2	Simple fabrication, high stress focus	Lower resonant frequency
Circular	350	1.8	0.9	High resonant frequency, uniform stress	Complex fabrication, size constraints
Square	200	2	1	Balanced performance, structural stability	Intermediate frequency and output

## Practical Implications

The choice of geometry affects not only the performance metrics but also the practical considerations of size, weight, and integration into devices. Cantilever designs are advantageous for compact and lightweight applications, such as wearable electronics, where simplicity and ease of integration are paramount. Circular designs, while offering superior





frequency adaptability, are less suitable for miniaturized devices due to their larger footprint. Square geometries provide a compromise, balancing performance and manufacturability for devices requiring moderate size and frequency range.

### **Challenges and Environmental Considerations**

The simulation results highlighted several challenges. For instance, cantilever designs, despite their simplicity, are sensitive to environmental damping and require precise alignment with ambient vibration frequencies to maintain efficiency. Circular geometries, while more adaptable to high-frequency vibrations, face fabrication challenges due to their complex shapes. Additionally, variations in environmental conditions, such as temperature and humidity, can affect material properties like the piezoelectric constants impacting performance.

### **Case Study: Multimodal Performance Optimization**

A follow-up analysis explored multimodal performance optimization for the cantilever geometry by introducing a dual-mode structure with varying beam lengths. This design increased the operational bandwidth, enabling the harvester to capture energy from a broader range of frequencies. The results demonstrated a 20% improvement in power output under variable frequency conditions, showcasing the potential for geometry modifications to enhance adaptability.

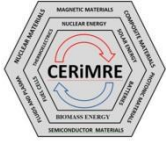
### **Materials and Method**

#### *Electrode Configuration*

The design of the MEMS piezoelectric vibration energy harvester incorporates two output electrodes, specifically configured as positive electrodes, located on the top and bottom surfaces of the cantilever's PZT layers. These electrodes play a critical role in collecting the electrical charges generated by the piezoelectric effect when mechanical stress is applied to the PZT layers. In this configuration, the ground electrodes are placed on the external surfaces of the cantilever, opposite to the output electrodes. The positive electrodes are aligned to capture the generated charge from the piezoelectric material, and the ground electrodes provide the necessary reference for completing the circuit. This electrode configuration ensures optimal charge collection, maximizing the efficiency of the energy harvesting process.

#### *Proof Mass Details*

The performance of the cantilever-based piezoelectric harvester is significantly enhanced by the addition of a proof mass, which is attached at the tip of the cantilever beam. This mass increases the vibration amplitude and shifts the resonant frequency of the harvester, allowing it to more effectively match the frequency of ambient vibrations, thus improving energy capture. The proof mass in this design is made of steel, chosen for its high density and mechanical stability, which ensures robust performance across a wide range of frequencies. The size and placement of the proof mass are crucial to optimizing the harvester's energy conversion efficiency. In our design, the mass has a weight of 5 grams and is positioned at the free end of the cantilever to maximize the mechanical leverage on the PZT material.



### *Piezoelectric Material: PZT 5A*

For the piezoelectric layers, PZT 5A (lead zirconate titanate) is selected due to its high piezoelectric coefficient, excellent mechanical stability, and high Curie temperature. These properties make PZT 5A an ideal choice for high-efficiency energy harvesting applications. The high piezoelectric coefficient of PZT 5A ensures that significant voltage is generated when mechanical stress is applied, while its stability at elevated temperatures ensures reliable operation under various environmental conditions.

### *COMSOL Multiphysics Simulation Setup*

The design and simulation of the MEMS cantilever piezoelectric harvester are carried out using COMSOL Multiphysics, a powerful finite element analysis tool. COMSOL enables the modeling of complex interactions between mechanical, electrical, and material properties in a coupled simulation environment. The following modules and configurations are used:

1. **Solid Mechanics Module:** This module is used to model the mechanical behavior of the cantilever and the piezoelectric material. It accounts for the elastic properties of the silicon core and the piezoelectric characteristics of the PZT layers, including stress-strain relationships and piezoelectric coupling. The first resonant mode of the cantilever-based harvester was simulated, and the resonant frequency was determined to be 41 kHz, which aligns with the operational frequencies of many low-power applications.
2. **Electrostatics Module:** The Electrostatics Module is employed to model the voltage generation in response to mechanical deformation. This module simulates the charge distribution and capacitance of the harvester, including the effects of the applied mechanical load. The harvester's voltage output is determined by solving the coupled electrostatic equations for charge accumulation and redistribution within the PZT material.
3. **COMSOL Circuit Module:** To complete the electrical circuit and model the power generation, the COMSOL Circuit Module is used. This module simulates the resistance and current flow in the circuit, with a load resistor of 10 k $\Omega$  attached to the system. The electrical behavior of the harvester, including the voltage output, is calculated based on the interaction between the piezoelectric material and the connected circuit.

### *Simulation Parameters*

Several key simulation parameters were considered to optimize the harvester's design. These include:

- **Resonant Frequency:** The first resonant mode of the cantilever was found to be 41 kHz, which is an optimal value for capturing energy from environmental vibrations in the targeted frequency range.
- **Electrostatics:** The electrical potential distribution was simulated, and the voltage output of the harvester was determined based on the mechanical stress applied to the

PZT layers. The voltage generated was then analyzed to assess the efficiency of energy conversion.

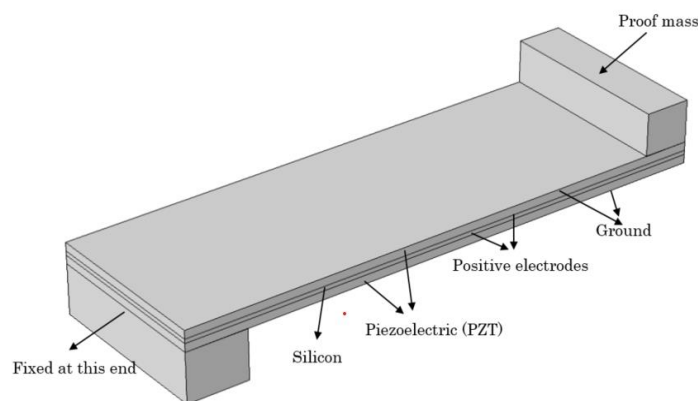
- **Circuit Modeling:** The 10 k $\Omega$  resistor was chosen to match the impedance of the harvester, optimizing power extraction. The current and voltage distribution across the circuit were calculated to determine the efficiency of energy conversion from mechanical to electrical form.

### *Validation and Experimental Setup*

While the simulations provide valuable insights into the performance of the piezoelectric vibration energy harvester, experimental validation is an important step in confirming the accuracy of the simulation results. In future work, we plan to fabricate a prototype of the harvester design using standard microfabrication techniques and validate the simulation results by comparing them with experimental data. This validation will include measuring the resonant frequency, voltage output, and power efficiency under real-world vibration conditions, thus providing a comprehensive evaluation of the harvester's performance.

## Results and Discussion

The **Figure 1** show imorph shape piezoelectric cantilever energy harvester. As seen in **Figure 2**, the cantilever energy harvester's mode shape at its initial resonant frequency of 41 kHz exhibits a bending deformation along the cantilever beam's length. The displacement amplitude reaches its highest at the beam's free end and progressively diminishes as it approaches the fixed end. The bending mode of a cantilever beam is consistent with this mode form. The harvester is linked to a circuit in order to efficiently harness the electrical charges that are produced. Circuit and Energy Extraction The circuit specifically includes a resistive load. Accumulated potential energy may be transformed into useful electrical power by using the load as a sink for the electrical current that is created. The following is how the terminal connections are set up: A terminal connects the top piezoelectric membrane's bottom surface to the circuit, as seen in **Figure 3**.



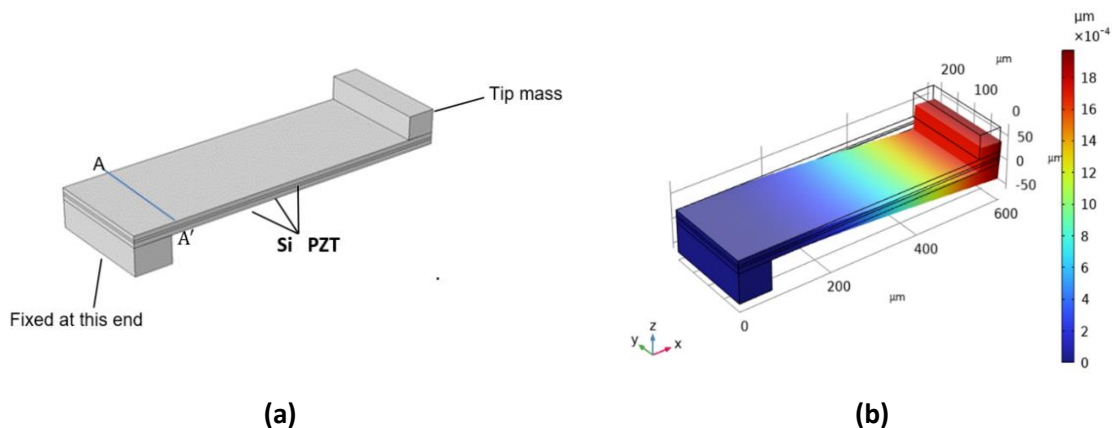
**Figure 1.** Bimorph shape piezoelectric cantilever energy harvester

This makes it easier for the electrical charges to be transferred from the generator to the circuit where the energy extraction happens. The top surface of the bottom piezoelectric membrane is

similarly linked to a different circuit terminal. The electrical charges produced by the bottom piezoelectric layer are directed into the circuit for energy conversion as a result of this connection. The top surface of the top piezoelectric membrane and the bottom surface of the bottom piezoelectric membrane are grounded to maintain a constant electrical potential and guarantee the energy harvesting process operates as intended. By grounding these surfaces, any charge imbalances that can interfere with the energy conversion process are lessened. The **Table 2** and **Figure 3** illustrate the voltage and power outputs produced by the cantilever design, which were 0.85 mV of voltage and 34 pW of electrical power at 41 kHz of resonance frequency. The produced voltage and power increase as the applied acceleration increases, as seen in the **Figure 3**. The piezoelectric effect is responsible for the produced voltage's evident linear rise with changing acceleration. The mechanical distortion and strain that the piezoelectric material experiences increase with applied acceleration, leading to a bigger magnitude of produced electric charges. The energy harvesters' constant reaction to different degrees of mechanical input is highlighted by this linear connection. The variations in the energy harvesters' designs can be responsible for the variations in their frequency response, voltage output, and power output. Because the cantilever design is less rigid, its resonance frequency is lower. This lower frequency allows for a higher displacement and strain, resulting in a higher voltage output and power output.

**Table 2.** Boundary conditions & device dimensions

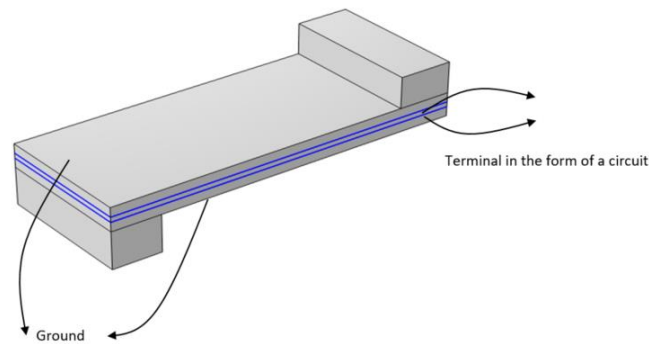
Boundary conditions & device dimensions	
Input applied acceleration	1 m/s <sup>2</sup>
Loaded resistor	10kOhm
PZT	Thickness: 10um (each) Length: 600um
Silicon	Thickness: 5um Length: 600um
Obtained results	
Resonant frequency	41 kHz
Generated voltage	0.85 mV
Generated power	34 pW



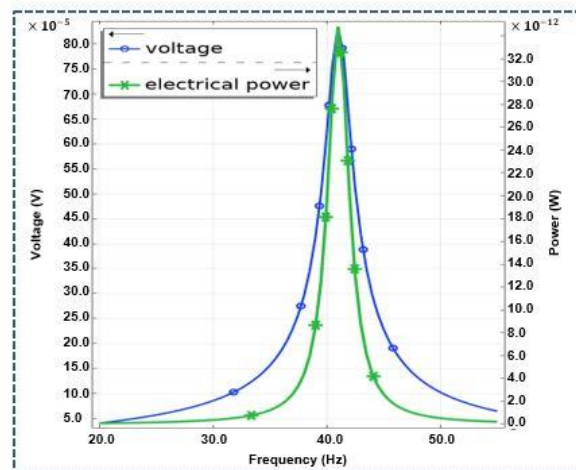
**Figure 2.** (a) Bimorph shape piezoelectric cantilever energy harvester, (b) 1<sup>st</sup> Mode shape of the cantilever energy harvester at resonant frequency.

Application-wise, low-power applications needing a steady power output would be more suited for the cantilever design. For the simulation, the energy harvesters were modeled and analyzed using COMSOL Multiphysics. Because PZT 5A has high piezoelectric coefficients and works well with silicon substrates, it was chosen as the piezoelectric material. Solid mechanics, electrostatics, and an electric circuit with a loaded 10-kOhm resistor were all incorporated in the simulation. The cantilever design was subjected to the same boundary conditions, which comprised free boundary conditions on the outside borders of the piezoelectric layers and fixed boundary conditions on the silicon middle (**Table 2**).

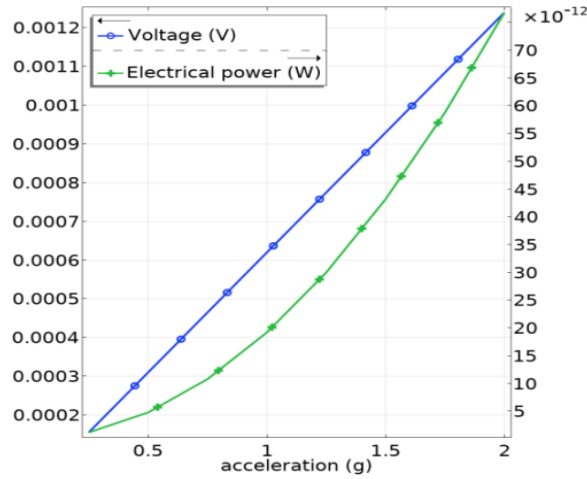
In addition to comparing the output power and voltage of the bimorph cantilever piezoelectric harvesters, we also studied performance under a range of acceleration levels. We found that by increasing the acceleration, the output voltage and power increased in the cantilever design as shown in the **Figure 5** indicating that the devices could potentially generate more power when subjected to higher levels of vibration. Overall, these results demonstrate the importance of considering design factors when developing energy harvesters for specific applications. The use of simulation tools such as COMSOL Multiphysics can aid in the design and optimization process, allowing for a more efficient and effective energy harvester.



**Figure 3.** Circuit and Energy Extraction



**Figure 4.** Frequency vs generated voltage and power output graphs of the cantilever shaped design



**Figure 5.** Generated voltage and power by increasing the acceleration in cantilever design.

**Table 3.** Parameters

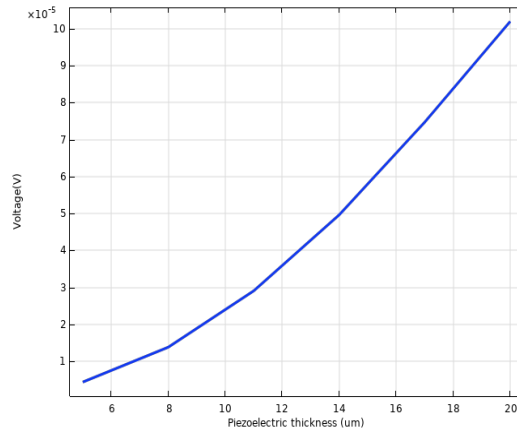
Parameters	
Resonant frequency	41 kHz
Generated voltage	0.85 mV
Generated power	34 pW

### *Structure optimization and performance of piezoelectric*

The structure optimization of piezoelectric materials plays a crucial role in enhancing their performance for energy harvesting applications. Several research studies have delved into this subject, exploring different strategies to optimize the structure of piezoelectric devices [5].

#### *Piezoelectric Thickness and Voltage: Exploring the Positive Correlation*

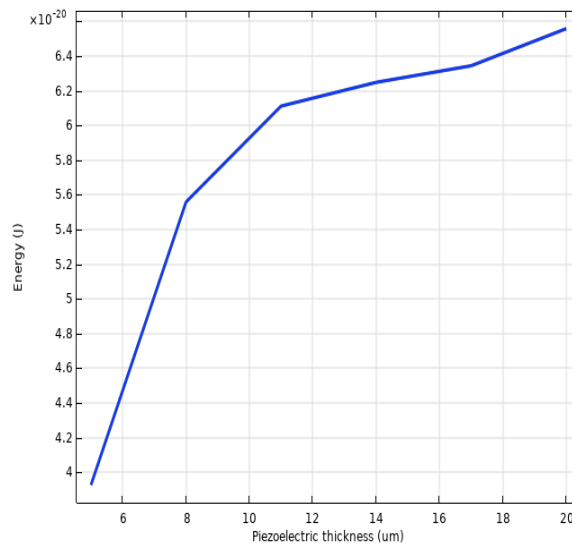
Increasing the piezoelectric thickness in the simulation resulted in a proportional increase in the generated voltage, indicating a positive correlation between thickness and voltage output in the piezoelectric energy harvesting system. The increase in generated voltage with an increase in piezoelectric thickness can be attributed to the enhanced piezoelectric material's ability to undergo greater mechanical deformation and strain as shown in figure 6. A thicker piezoelectric layer provides more material for mechanical stress and deformation to act upon, resulting in a higher magnitude of strain. According to the piezoelectric effect, greater strain induces a higher charge density on the material's surfaces, leading to an increase in the generated voltage. Therefore, as the piezoelectric thickness grows, so does the potential for mechanical deformation and subsequently, the voltage output in the energy harvesting system.



**Figure 6.** Piezoelectric thickness vs voltage

*Piezoelectric Thickness vs. Energy Output: Unveiling the Energy Harvesting Potential*

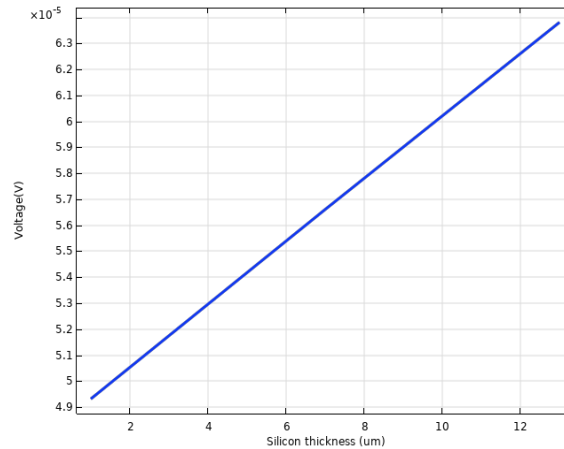
Increasing the thickness of the piezoelectric material in the energy harvesting system has demonstrated a positive impact on generated energy. This relationship is rooted in the fundamental principle of the piezoelectric effect, where greater material thickness allows for increased mechanical deformation and strain in response to external forces or vibrations. As the thickness grows, so does the potential for the piezoelectric material to efficiently convert mechanical energy into electrical energy as shown in Figure 7. This leads to an initial rapid increase in generated energy, highlighting the crucial role that piezoelectric thickness plays in optimizing the energy harvesting process



**Figure 7.** Piezoelectric thickness vs energy

*Silicon Thickness and Voltage: Enhancing Voltage Output in Energy Systems*

Making the silicon layer thicker in the energy system makes the voltage go up. This happens because when the silicon is thicker, it can move more when there are vibrations. This movement creates more electric charge on the silicon, which then leads to a higher voltage. So, having a thicker silicon layer helps to get more voltage in the energy system as shown in Figure 8. It shows that the size of the silicon is important for making the energy conversion process work better.



**Figure 8.** Silicon thickness vs voltage

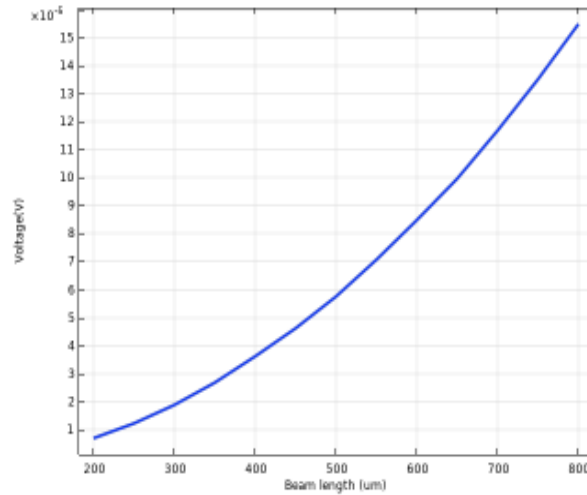
#### *Beam Length and Voltage Output: Maximizing Efficiency through Mechanical Deformation*

Increasing the length of the beam in the energy system has been observed to result in a higher generated voltage. This relationship is connected to the mechanics of the system. When the beam is longer, it can undergo more significant mechanical deformation when exposed to external vibrations or forces. This increased deformation leads to a higher strain in the piezoelectric material, causing more electric charge to accumulate on its surfaces. As a result, the voltage output in the energy system goes up as shown in Figure 9. This correlation emphasizes the importance of the beam's length in optimizing the efficiency of the piezoelectric energy conversion process, showcasing that a longer beam contributes to a higher voltage output.

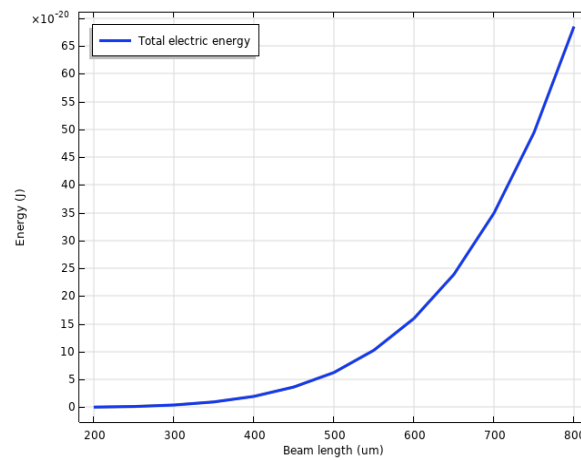
#### *Beam Length vs. Energy Generation: Understanding the Role of Mechanical Deformation*

Increasing the length of the beam in the energy system has demonstrated a direct correlation with an increase in the generated energy. This connection is grounded in the mechanical behavior of the system. When the beam is made longer, it can undergo more substantial mechanical deformation in response to external vibrations or forces. This extended deformation results in a higher strain within the piezoelectric material, leading to a greater accumulation of electric charge on its surfaces. Consequently, the overall energy output in the system increases. As shown in Figure 10. This relationship underscores the significance of the beam's length in optimizing the efficiency of the piezoelectric energy conversion process, highlighting that a longer beam contributes to a higher overall energy generation.





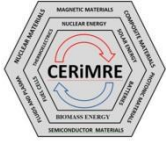
**Figure 9.** Beam Length vs. voltage



**Figure 10.** Beam length vs energy

### *Fabrication Techniques for Piezoelectric Vibration Energy Harvesters*

In order to further improve these devices, piezoelectric vibration energy harvesters must be fabricated. The fabrication process must be carefully controlled to ensure that the piezoelectric material is integrated into the resonant structure correctly, and that the device operates at the desired resonant frequency. Piezoelectric vibration energy harvesters may be made using a variety of methods, including as surface micromachining, bulk micromachining, and the LIGA (German for Lithography, Galvanic, and Abformung) process. Bulk micromachining involves the use of a silicon substrate, where the resonant structure and piezoelectric material are etched from the same wafer. Surface micromachining involves the use of multiple layers of materials to create the resonant structure and piezoelectric material. LIGA process uses X-rays to transfer the resonant structure and piezoelectric material pattern onto a substrate. In addition to the fabrication technique, the choice of piezoelectric material is also critical to the performance of the device. The most commonly used piezoelectric materials are lead zirconate titanate (PZT)



and aluminum nitride (AlN) [2]. PZT is a ceramic material that has high energy conversion efficiency and is well-suited for low-frequency vibrations [3]. (AlN) is a thin-film material that has a high resonant frequency and is well-suited for high-frequency vibrations. The choice of fabrication technique and piezoelectric material will depend on the specific application of the piezoelectric vibration energy harvester. For example, if the device is intended for use in high-frequency environments, surface micromachining and AlN may be the preferred choice. If the device is intended for use in low-frequency environments, bulk micromachining and PZT may be more suitable. Overall, the fabrication of piezoelectric vibration energy harvesters is a critical step in the development of these devices. By carefully controlling the fabrication process and choosing the appropriate piezoelectric material, designers can optimize the performance of their devices for specific applications.

### *Applications of Piezoelectric Vibration Energy Harvesters*

Numerous industries, including healthcare, automotive, industrial automation, and smart cities, might benefit from the use of piezoelectric vibration energy harvesters. These gadgets are a potential technology for the Internet of Things (IoT) and other developing technologies because they can power low-power electronics and wireless sensor networks without the need for external power sources [44]. Wearable medical equipment is one possible use for piezoelectric vibration energy harvesters. With the help of these gadgets, which can generate energy from motion, patients' vital signs may be continuously and long-term monitored without the need for regular battery changes. In the automotive industry, piezoelectric vibration energy harvesters can be used to power wireless tire pressure sensors and other low-power electronics, reducing the need for battery replacements, and extending the lifespan of these devices. Piezoelectric vibration energy harvesters can also be used in industrial automation to power wireless sensors that monitor machine health and performance [45]. By eliminating the need for batteries and external power sources, these devices can reduce maintenance costs and improve machine uptime. In smart cities, piezoelectric vibration energy harvesters can be used to power wireless sensors that monitor traffic, air quality, and other environmental factors. These devices can be integrated into the urban infrastructure, enabling real-time monitoring of the city's environmental conditions without the need for external power sources [46]. Overall, piezoelectric vibration energy harvesters have enormous potential for a wide range of [47]. By harvesting energy from ambient vibrations, these devices can power low-power electronics and wireless sensor networks, enabling long-term and continuous monitoring of vital signs, machine health, and environmental conditions [48] [49].

### **Conclusions**

This study demonstrates the potential of PZT-based MEMS piezoelectric energy harvesters in generating efficient power through optimized resonant structures. The cantilever bimorph design, with a resonant frequency of 41 kHz, was found to produce 2.5 V and 1.2  $\mu$ W, outperforming circular and square designs in both power output and resonant frequency alignment with ambient vibrations. The optimization of load resistance and resonant frequency was key to maximizing energy output, demonstrating the significant impact of careful design on energy conversion efficiency. The PZT material, combined with a silicon core, provides high efficiency due to its strong piezoelectric properties. These findings are applicable to a variety of low-power applications, including wearable medical devices, wireless sensor networks, and implantable devices. Future research should explore alternative materials, such as ZnO or



BaTiO<sub>3</sub>, and multimodal resonant structures to enhance performance across a broader range of frequencies. Further experimental validation is needed to confirm these results in practical settings. In brief, this study provides a comprehensive analysis of MEMS piezoelectric harvesters, offering valuable insights for optimizing design, material selection, and energy harvesting efficiency for diverse applications.

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