**Computational and Experimental Research in Materials and Renewable Energy (CERiMRE)**
**Volume 5, Issue 1, page 56-62**
**eISSN : 2747-173X**

Submitted: April 10, 2022
Accepted: May 25, 2022
Online: May 31, 2022
DOI: 10.19184/cerimre.v5i1.31566

# Neutron Mean Free Path in the Slab Reactor Core using One-Dimensional Multi-group Diffusion Equation

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**Abstract.** Analysis of the neutron mean free path in the slab reactor core has been carried out using one-dimensional multi-group diffusion equation. This study aims to determine the neutron mean free path in the slab reactor core with the neutron diffusion coefficient calculation using macroscopic cross-section data in the nuclear fuel cell level and the neutron flux distribution. The type of reactor used in this research is a fast reactor with nuclear fuel is uranium-plutonium nitride (U-PuN). The neutron mean free path is calculated for 70 energy groups of neutrons by dividing the energy groups, namely the fast energy group, the intermediate energy group and the thermal energy group. The results showed that the neutron mean free path value for U-235 and Pu-239 fuels were obtained almost the same in all energy groups, namely in the fast energy group ranging from 0.11.10-2 to 0.17.10-2 cm, in the intermediate energy group 0.16.10-2 to 1.78.10-2 cm, and in the thermal energy group 0.4.0-2 to 8.04.10-2 cm. The neutron mean free path value for U-238 fuel is much smaller than that for U-235 and Pu-239 fuel, ranging from 0.03.10-2 to 0.36.10-2 cm. These results can be confirmed, because U-238 fuel is a fertile material.

**Keywords:** Neutron mean free path, Diffusion equation, Neutron flux, Slab reactor core.

## Introduction

Neutronic analysis is the fundamental part in studying nuclear reactor systems, besides the problems of thermal hydraulics and reactor safety. Neutron transport is described as an integro-differential transport equation with energy, space and time variables [1]. Neutron transport is very important to solve because the distribution of neutrons in the reactor core is related to the distribution of reactor power. In order to properly design a nuclear reactor, it is necessary to predict how the neutrons are distributed throughout the system. Unfortunately, determining the distribution of neutrons is a difficult problem in general [2]. The ideal neutron flux distribution will be achieved if the neutron flux is perfect, i.e. the average neutron flux is equal to the maximum neutron flux. However, a perfectly neutron flux distribution is difficult to achieve. One way to obtain the ideal neutron flux without increasing the maximum flux is to calculate the neutron mean free path. The neutron mean free path is the distance traveled by a neutron before colliding with a nuclide [3]. The neutron mean free path determines how far the neutron travels before colliding the nucleus and participating in the type of reaction that is formed. In the theory, materials with high absorption have short the neutron mean free path values. In the calculation, the neutron mean free path value is influenced by the value of the macroscopic cross-section, the distribution of the neutron flux and the neutron diffusion coefficient. The calculation of the neutron mean free path value in this study can be compared with theory and find out how neutrons interact with matter.

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Solving the diffusion equation gives the shape of the distribution of the neutron flux with respect to space and energy. In the diffusion equation, the neutron energy is assumed to have energy groups, so the equation is called the multi-group diffusion equation. Shafii *et.al*. [4] investigated the value of the neutron diffusion coefficient as a function of energy using multi-group diffusion equation with a macroscopic cross-section value as the input. The results show that the value of the diffusion coefficient to the extrapolated distance is only accurate in the fast energy group, and the value of the diffusion coefficient to the energy function is greater in fissile materials than in fertile materials.

The macroscopic cross-section data required from this study refers to the results of research from Shafii *et.al.* [5]. According to [5], from the results of the homogenization of nuclear fuel cells in a fast reactor with uranium-plutonium nitride (U-PuN) fuel and lead-bismuth (Pb-Bi) as a coolant, the total macroscopic cross-section for uranium and plutonium nuclides undergoing overlap in the high energy region, giving results that are in accordance with the reference, namely fast neutrons reacting at high energies.

In contrast to previous studies, this study determine the neutron mean free path in the reactor core in the form of a one-dimensional slab using the multi-group diffusion equation. The slab reactor core is assumed to be composed of homogeneous nuclear fuel cells. This research is part of the continued neutronic analysis, which is to determine the mean free path of the neutrons in the fuel cell as a function of energy. This research uses a fast reactor type design and uses uranium-plutonium nitride (U-PuN) as fuel and lead-bismuth (Pb-Bi) as coolant. This research begins by calculating the neutron flux at the fuel cell level. The neutron mean free path calculation will complement the neutronic analysis data in the design of nuclear reactors that will be better. This research is in the form of developing a nuclear computing program using Delphi programming.

## Theoretical Background

At steady state and one-dimensional homogeneous medium, the neutron diffusion equation can be written as,

$$-D_g\nabla^2\phi_g(x)+\Sigma_{ag}\phi_g(x)=S_g(x) \tag{1}$$

where g is an index of neutron energy group, $D_g$ is neutron diffusion coefficient, $\phi_g$ is nutron flux, $\Sigma_{ag}$ is an absorption macroscopic cross-section and $S_g$ is neuron source. Equation (1) can be simplified to

$$\frac{d^2\phi_g(x)}{dx^2}-\frac{\Sigma_{ag}}{D_g}\phi_g(x)=-\frac{S_g(x)}{D_g} \tag{2}$$

The boundary conditions that apply to equation (2) are

$$\phi(x=0)=\phi(x=L)=0 \tag{3}$$

$$S(x=0)=S(x=L)=0 \tag{4}$$

Equation (2) can be written in discrete form as follows:

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$$\frac{\phi_{(i+1)g} - 2\phi_{ig} + \phi_{(i-1)g}}{(\Delta x)^2} - \frac{\Sigma_{ag}}{D_g}\phi_{ig} = -\frac{S_{ig}}{D_g} \quad (5)$$

The neutron flux can be obtained from equation (5) using Jacobi method

$$\phi_{ig}^{new} = \frac{\frac{\phi_{(i+1)g}^{old}+\phi_{(i-1)g}^{old}}{(\Delta x)^2} + \frac{S_{ig}}{D_g}}{\frac{\Sigma_{ag}}{D_g} + \frac{2}{(\Delta x)^2}} \quad (6)$$

Thus, the diffusion coefficient can be obtained as follows:

$$D_g = \frac{(\Delta x)^2\left(\phi_{ig}^{new}\Sigma_{\mathrm{ag}} - \mathrm{S_{ig}}\right)}{\phi_{(i+1)g}^{old} + \phi_{(i-1)g}^{old} - 2\phi_{ig}^{new}} \quad (7)$$

The neutron mean free path is obtained from the neutron diffusion coefficient according to Fick's law

$$D_g = \frac{1}{3}\lambda_{trg} \quad (8)$$

so that it is obtained the neutron mean free path $\lambda_{trg}$ as a function of group energy

$$\lambda_{trg} = 3\left[\frac{(\Delta x)^2\left(\phi_{ig}^{new}\Sigma_{\mathrm{ag}} - \mathrm{S_{ig}}\right)}{\phi_{(i+1)g}^{old} + \phi_{(i-1)g}^{old} - 2\phi_{ig}^{new}}\right] \quad (9)$$

The equation (9) will be used in computational calculations to obtain the neutron mean free path.

## Materials and Methods

*Reactor Core Design Specification*

Figure 1 shows the geometric design of the finite slab reactor core with a distance on the x-axis from 0 to L, which is 20 cm. In this study, the 1D diffusion equation to calculate the neutron flux is only in the x-direction, so that the reactor core height can be neglected. The input values used are the macroscopic cross-sections obtained by [5] and the neutron flux distribution by [6] which are then used to determine the neutron mean free path.

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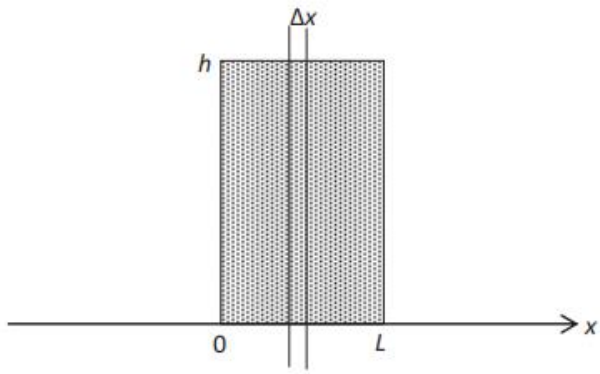


Figure 1. Design of finite slab reactor core with height *h* and width *L*

*Computational Procedure*

To facilitate the computational calculations, this study uses various approaches [4]:

1. The cross section of nuclear fuel elements in each region is constant.
2. The flux at any volume is constant, this assumption is referred to the flat flux approximation.
3. The source in each volume is constant, this assumption is called the flat source approximation.
4. The flat source approach is only appropriate if the external source is in the reactor core.
5. The rates of fission and scattering reactions are set as source terms.
6. The reactor core is in the form of a slab and is assumed to be composed of homogeneous nuclear fuel cells.

In the slab reactor core, usually the dominant nuclear fuel is U-235 with the slab width is considered L, but in this study three fuels were used, namely U-235, U-238 and Pu-239. Furthermore, the computation procedure for calculating the mean free path is carried out as follows:

1. The initial neutron source boundary conditions and the neutron flux are determined.
2. Macroscopic cross-section of U-235, U-238 and Pu-239 are arranged in a homogeneous region and is taken from a reference [7].
3. The presence of an external source in the system is determined.
4. Neutron diffusion coefficient and neutron flux are taken from the reference [5,6].
5. Spatial variable of the mesh is defined.
6. Calculation of the mean free path using equation (9)
7. The procedure was repeated for different neutron energy groups, varying from energy group 1 to 70 for each fuel.

## Results and Discussion

The neutron mean free path values for U-235 fuel can be seen in Figure 2. In the fast energy group, the neutron mean free path values of U-235 range from $0.11.10^{-2}$ cm to $0.17.10^{-2}$ cm. This happens because U-235 is a fissile material, fissile material in the cross-section pattern of fission reactions decreases in the fast energy group [7]. In the fast energy group, there is no medium

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that slows down the motion of the neutrons, so the value of the diffusion coefficient is much smaller [5]. Judging from the macroscopic cross-section value, the absorption is small in the fast energy group and the value of the neutron flux distribution is almost uniform in all groups. This condision causes the neutron mean free path value of U-235 to be the smallest in the fast energy group. Besides that, the U-235 fuel is also a fissile material that can fission when fired by neutrons in the all energy group. In the intermediate energy group, the neutron mean free path value experienced resonance, i.e. the value is fluctuating in each energy group in a fairly close energy range. This occurs because the interaction of neutrons with matter in the intermediate energy group is unstable. In the thermal energy group, there is more absorption of neutrons in the nucleus, which is called an absorption reaction. The value of the flux distribution in the thermal energy group is also smaller than the value in the other energy groups. This is what causes the neutron mean free path value is largest in the thermal energy group. In this situation, the neutrons interacting with the material U-235 experience accurate collisions with the corresponding neutron mean free path values for each energy group.

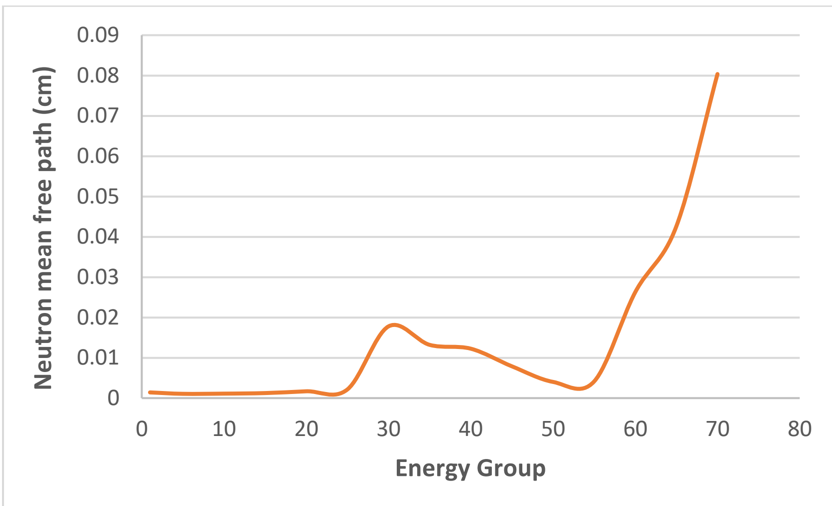


Figure 2. The neutron mean free path of U-235

The neutron mean free path value obtained from the calculation results for Pu-239 is a low value in the fast energy group, medium in the intermediate energy group and high in the thermal energy group, as shown in Figure 3. Pu-239 is a fissile fuel as well as U-235 so the neutron mean free path value is not much different, where the neutrons that interact with the fissile material will undergo a fission reaction process. Pu-239 absorbs thermal neutrons to form a compound nucleus which will be excited to a higher energy level than the critical energy. In the thermal energy group of Pu-239, more neutron absorption occurs in the nucleus. This is what causes the largest neutron mean free path value in the thermal energy group.

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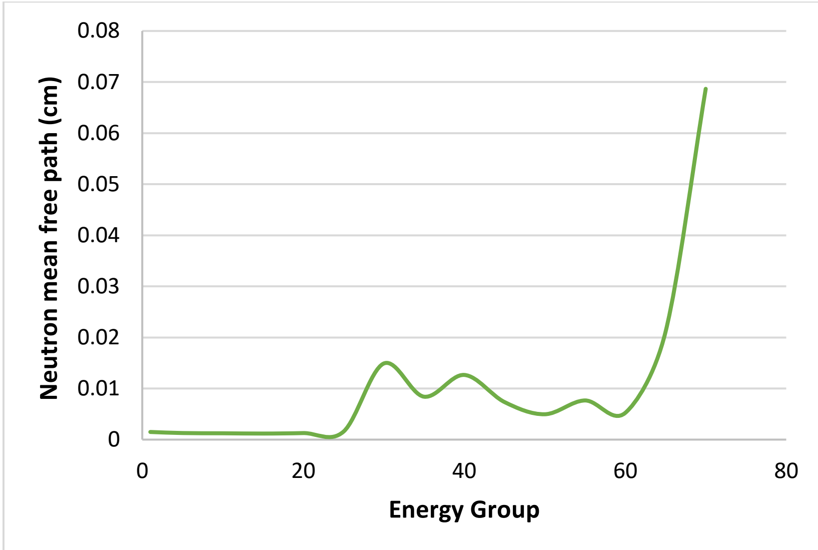


Figure 3. The neutron mean free path of Pu-239

The neutron mean free path value of U-238 is much smaller than U-235 and Pu-239, which is only in the range of $0.03.10^{-2}$ cm to $0.36.10^{-2}$ cm. This condition can be understood because U-238 is a fertile fuel, where the properties of this material are different from fissile fuels [8]. From the Figure 4, it can be seen that the neutron mean free path values is high in the intermediate and the thermal energy group, however in the fast energy group obtained a much smaller value. Besides being influenced by the input value of the macroscopic cross-section and the neutron diffusion coefficient, the neutron mean free path value obtained at U-238 is also due to the nature of U-238 which is a fertile material. The interaction of neutrons with fertile material will usually form a reaction that converts the fertile material into fissile material first, so the value is much smaller than the interaction of neutrons with fissile material.

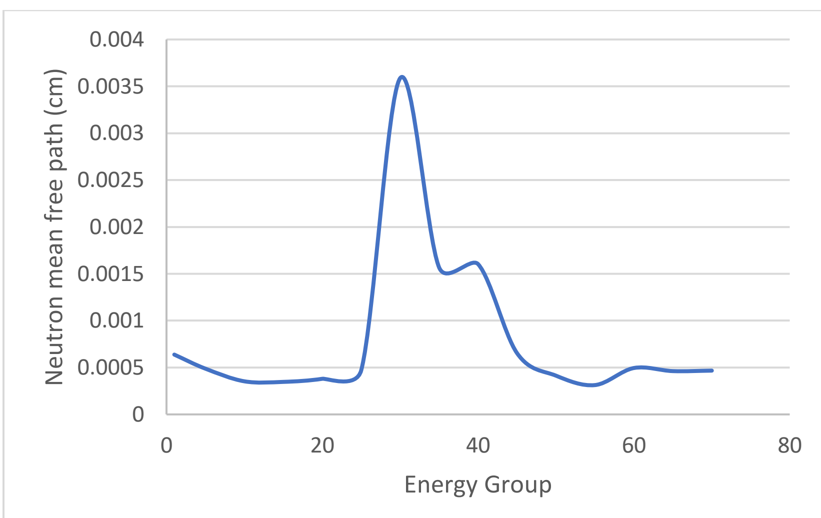


Figure 4. The neutron mean free path of U-238

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## Conclusions

Based on the calculations and analyzes that have been carried out, the neutron mean free path in the slab reactor is influenced by the macroscopic cross-section value, the distribution of the neutron flux and the neutron diffusion coefficient where the neutron mean free path in the fissile material of U-235 and Pu-239 is greater than in the fertile material of U-238. The neutron mean free path value is higher in the thermal energy group and smaller in the fast energy group for each fuel due to the effect of neutron interactions on each material. The interaction of neutrons with the nuclide was obtained faster in the fast energy group compared to other, which one the neutron mean free path value did not reach to 1 cm. This is because the reactor design chosen in this study is a fast reactor type.

**ACKNOWLEDGEMENTS**

This research is funded by Directorate of Resources, Directorate General of Higher Education, Ministry of Education, Culture, Research and Technology, Republic of Indonesia, according to the Research Contract Number: 021/E4.1/AK.04.PT/2021 and LPPM Unand Number: T/10/UN.16.17/PT.01.03/PD-Energi/2021.

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